GEOLOGY 220: TECTONICS
Homework 2: Isostasy

Introduction

The study of plate tectonics usually emphasizes horizontal motions of plates, but vertical dynamics are intimately related to the horizontal motions. Vertical motions have some very important consequences - subduction, mountains building, basins development, weather pattern alteration, erosion rate acceleration, and exposure of deep rocks, to name a few. In this problem set, you will explore the vertical dynamics of the lithosphere through the concept of isostasy.

Some background on isostasy

Isostasy is an equilibrium condition characterized by equal pressure within some fluid. In the case of plate tectonics, the fluid happens to be the mantle. (NB! Do not confuse the word *fluid* here to mean that the mantle is a liquid – solid rock can be a fluid provided that it can flow in response to stresses. It's just that this flow occurs over geologic time scales.) For the pressure in the mantle to be equal at a given depth, there must be an equal amount of overlying mass everywhere above that depth.

Imagine that we take a particular depth within the mantle to call our compensation depth. We then define a series of vertical columns rising from that depth to sea level. If the mass of each of these columns is the same, then we have a state of isostatic equilibrium; any deviation from this will provoke an isostatic adjustment in the form of a vertical motion whose speed is determined by the viscosity of the mantle and the magnitude of the pressure difference within the mantle.

For example, if one column is deficient in mass, then the mantle in that region is at lower than normal pressure, and this initiates flow in the mantle to fill in this low pressure area. More mantle material is thus added to the column, causing upward motion of the surface. If a column has too much mass, the mantle below is at a higher than normal pressure, so mantle material moves away from that region, causing the surface to drop in elevation. This concept of isostasy can be applied to a number of questions. For instance, we can figure out the thickness of oceanic crust as shown in the example below, by equating two lithospheric columns in isostatic equilibrium.

Example: Calculating the thickness of the oceanic crust

If we say that these two columns of rock in Figure 1 are in isostatic equilibrium, then they are neither rising nor subsiding, and the masses of these two columns must be equal. To determine the thickness of oceanic crust in this example, we therefore need to create an equation relating the masses of these two columns ($m_{column1} = mass_{column2}$) and solve for the unknown we desire (i.e. h_{oc}).

We can simplify our calculations assuming that for each column, the width and depth are both equal to one unit. Our simplification allows us to say that the volume of each segment of the rock column is equal to that segment's height and we can then use the following equation:

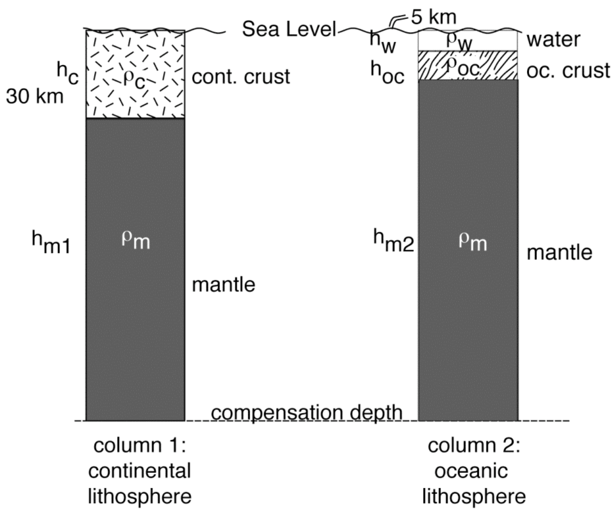

Figure 1: two columns in isostatic equilibrium

(1) $h_c\rho_c + h_{m1}\rho_m = h_w\rho_w + h_{oc}\rho_{oc} + h_{m2}\rho_m$.

The trick is that there are 2 unknowns here (h_{oc} and h_{m2}) but only one equation. We need to create a second equation and solve the two equations simultaneously in order to determine the oceanic crustal thickness. What other piece of information do we know? Well, we know that the height of each rock column above the compensation depth is the same (sea level). This gives us:

(2) $h_c + h_{m1} = h_w + h_{oc} + h_{m2}$.

If we rearrange this equation to solve for h_{m2} (one of the two unknowns in the equation (1)), we can plug this new version of equation (2) into equation (1), yielding:

(3) $h_c\rho_c + h_{m1}\rho_m = h_w\rho_w + h_{oc}\rho_{oc} + (h_c + h_{m1} - h_w - h_{oc})\rho_m$.

Simplifying and rearranging this equation, we can now find an expression for the oceanic crustal thickness:

(4) $h_{oc} = \dfrac{h_c(\rho_c - \rho_m) + h_w(\rho_m - \rho_w)}{\rho_{oc} - \rho_m}$.

Plugging in realistic values for the different parameters including h_c = 30 km, h_w = 5 km, ρ_c = 2.83 g/cm^3, ρ_{oc} = 2.95 g/cm^3, ρ_m = 3.3 g/cm^3, and ρ_w = 1.0 g/cm^3, we find that h_{oc} = 7.4 km.

In this example, we didn't insert values until the equation was solved symbolically. Although you could plug in values sooner in the solution process, symbolically solving the system of equations allowed us to cancel terms and end up with a fairly simple expression for h_{oc}. An even bigger advantage is that now we can change values of thicknesses and densities and easily recalculate crustal thicknesses.

These kinds of problems always boil down to the same thing: you draw two lithospheric columns and assume they are in isostatic equilibrium, which means that their masses are the same. If there is just one unknown in this equation, then you're all set. If there are two unknowns, you need another equation. Typically, you can also assume that the heights of the columns are the same, and this gives you two equations. Combining your two equations allows you to solve the problem.

Assignment

Following a similar method as the one outlined in the example, solve the following problems. About your solutions: Draw a sketch first to help yourself organize the problem. Turn in neat solutions – this may mean that you need to work out solutions on scratch paper first. Please write your solutions in complete sentences. Note that mathematical equations are actually English but be sure to include other helpful words, phrases or sentences to show how you got from one step to the next. So…don't just give me a list of equations and expect me to follow what you did. Use the example problem as a guide if you are unsure. Last, be careful - it is really easy to make algebra mistakes when solving these problems, so use your intuition to help you tell if you've got major errors. (Ask yourself – does my answer make sense?)

1. Imagine that you fill the ocean, depicted in the right column in Figure 1, with sediment in an instant. The total thickness of sediment dumped in is equal to the water depth, h_w.

a. Do an isostatic calculation to determine the water depth after the crust gets back into isostatic equilibrium. (It'd be out of equilibrium just after you dump the sediment in). Assume the density of sediment is 2.3 g/cm^3. Draw the two columns of equivalent mass and show your calculations. Don't worry about the displaced water. The ocean has a huge area to hold that water without changing sea level.

b. What is the maximum amount of sediment that could accumulate in the above case without the sediment rising above sea level? Again, draw the two columns of equivalent mass and show your calculations.

2. When mountains erode, material is removed from the surface and this has an isostatic effect. Imagine you have a column with continental crust of 60 km thickness.

a. If continental crust with a thickness of 30 km is right at sea level, what would be the elevation (above sea level) of the mountain tops in the case of the 60 km thick crust? The 30 km crustal column is your reference column. Solve for the height above sea level of the other column.

b. Now imagine that you erode 2 km of sediment off the mountain tops in an instant. What will the average mountain elevation be after isostatic equilibrium is restored? What happens to the base of the crust during this isostatic adjustment? If you can only erode when the surface elevation is above sea level, what will the ending thickness of the crust be (assuming you allow erosion to go on and on as long as it can)?