ERRATA TO "SETTLED POLYNOMIALS OVER FINITE FIELDS"

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All results in the paper [1] are valid when the polynomials involved are monic. To make them so for general polynomials, we change the following.

We define the adjusted critical orbit to depend on the leading coefficient of the quadratic polynomial f. The sentence just before Proposition 2.3 should read: "To state this criterion, we need some terminology: the *critical orbit* of a quadratic $f \in K[x]$ with leading coefficient a is the set $\{f^i(\gamma) : i = 2, 3, \ldots\}$, where γ is the critical point of f, while the *adjusted critical orbit* of f is $\{-af(\gamma)\} \cup \{af^i(\gamma) : i = 2, 3, \ldots\}$."

We also replace Lemma 2.5 with the following:

Lemma 2.5'. *Let K be a field of characteristic not equal to two, let $f(x) = ax^2 + bx + c \in K[x]$, and let $\gamma = -b/2a$ be the unique critical point of f. Suppose that $g \in K[x]$ has leading coefficient $\ell(g)$ and degree $d \geq 1$, and suppose that $g \circ f^{n-1}$ is irreducible over K for some $n \geq 1$. Let $C = (-a)^d \ell(g)$ if $n = 1$ and $C = a^d \ell(g)$ if $n \geq 2$. Then $g \circ f^n$ is irreducible over K if $Cg(f^n(\gamma))$ is not a square in K. If K is finite then we may replace "if" with "if and only if."*

Proof. By Capelli's Lemma and the irreducibility of $g \circ f^{n-1}$, we have $g \circ f^n$ irreducible if and only if for any root β of $g \circ f^{n-1}$, $\mathrm{Disc}\,(f(x) - \beta) = b^2 - 4ac + 4a\beta$ is not a square in $K(\beta)$. Denote by $\ell(g \circ f^{n-1})$ the leading coefficient of $g \circ f^{n-1}$. Then

$$\begin{aligned}
N_{K(\beta)/K}(b^2 - 4ac + 4a\beta) &= (-4a)^{\deg(g \circ f^{n-1})} \prod_{\beta \text{ root of } g \circ f^{n-1}} \left(-\frac{b^2}{4a} + c - \beta\right) \\
&= (-4a)^{d2^{n-1}} \cdot \ell(g \circ f^{n-1})^{-1} \cdot (g \circ f^{n-1})(-b^2/4a + c) \\
&= (-4a)^{d2^{n-1}} \cdot a^{-d(2^{n-1}-1)} \cdot \ell(g)^{-1} \cdot (g \circ f^{n-1})(f(\gamma)).
\end{aligned}$$

This proves the Lemma in the case of a general field K. Note that $N_{K(\beta)/K}$ is a multiplicative homomorphism, thus mapping squares to squares. If $K = \mathbb{F}_q$ then since $1/2$ of the elements of $\mathbb{F}_q(\beta)^*$ are squares, it follows that $\alpha \in \mathbb{F}_q(\beta)$ is a square if and only if $N_{\mathbb{F}_q(\beta)/\mathbb{F}_q}(\alpha)$ is a square in $\mathbb{F}_q$. $\square$

Finally, throughout Section 3 we add the hypothesis that all polynomials are assumed monic.

References

[1] Rafe Jones and Nigel Boston. Settled polynomials over finite fields. *Proc. Amer. Math. Soc.*, 140(6):1849–1863, 2012.

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