Enlightening Symbols: A Short History of Mathematical Notation by Joseph Mazur

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Enlightening Symbols: A Short History of Mathematical Notation

by Joseph Mazur

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REVIEWED BY RAFE JONES

A mathematically minded person can hardly imagine a world without the symbols that form the connective tissue of mathematical statements. So woven are they into the fabric of our mental processes that many of them come prewritten on any keyboard: +, −, =, <, and the symbols 1 through 9, just to name a few. This doesn't even touch on familiar algebraic representations such as letters to denote unknown quantities, superscript numerals to denote powers, radicals to denote roots, and functions written as f(x). It was not always so. Until as recently as the sixteenth century, not only did most mathematical expressions contain no symbols recognizable to us today, but they were *purely rhetorical*. In his entertaining and quirky *Enlightening Symbols: A Short History of Mathematical Notation*, Joseph Mazur drives home how this transformation has shaped our mathematical consciousness by quoting the following translation of a passage from al-Khwarizmi's *Algebra* (ca. 820 AD):

> If a person puts such a question to you as: "I have divided ten into two parts, and multiplying one of these by the other the result was twenty-one"; then you know that one of the parts is thing, and the other is ten minus thing (p. xv).

In modern notation, this is $x(10 - x) = 21$. In addition to its compactness, the modern expression allows us to focus on the essential quantities and to ask key questions more easily: what if we changed the values 10 and 21? Is there a general method for solving all such problems? Mazur, formerly on the mathematics faculty at Marlboro College and a self-described "mathematics journalist embedded with the learned troupes" (p. 233), is careful to note that the passage mentioned earlier represents an extreme, and that some authors used positional Hindu-Arabic numerals and denoted unknown quantities with letters well before the sixteenth century. Nonetheless, I could not help but be shocked at the recentness of our seemingly timeless way of writing mathematics.

Through a series of short chapters that read as vignettes, Mazur recounts the advent of mathematical symbols starting from the very beginning—cave paintings 40,000 years ago—and ending with the calculus symbols introduced by Leibniz and Newton. In between, I found a smorgasbord of captivating observations, reflections, anecdotes, historical narration, and quotations (the last from a dizzying array of personages, including historians, mathematicians, philosophers, psychologists, and novelists). For example, I learned that in his marvelously titled 1557 work, *Whetstone of Witte*, Robert Recorde introduced the twin lines of the symbol =, calling them "Gemini lines," to denote that the quantities on either side are as identical as possible, and invoking the mythological Gemini twins, Castor and Pollux (p. 133). I also learned that Cardano, famous for his publication of the general solution of the cubic polynomial, somehow found room in his mind to accommodate nonreal complex numbers, but not the notion that the product of two negative numbers must be positive (p. 123).

In addition to the aforementioned smorgasbord, I also found puzzling loose ends, hard-to-follow chronology, and odd repetitions. This is a great book to pick up when you have half an hour to spare—at almost every turn there is something interesting and unexpected—and a more frustrating book to read straight through. Read it to whet your appetite for mathematical history, to spur ruminations on what mathematical notation accomplishes, and to add to your store of tidbits and anecdotes to tell to students or to other mathematically interested people. Don't read it to obtain a comprehensive and clear account of the historical development of modern numerals and other mathematical notation. For that I recommend Karl Menninger's *Number Words and Number Symbols* (or for a more modern and comparative study of number systems, Stephen Chrisomalis's *Numerical Notation*) and Florian Cajori's *A History of Mathematical Notations*.

The book is divided into three parts, titled "Numerals," "Algebra," and "The Power of Symbols." The first two revolve, respectively, around the historical development of positional representation of numbers via Hindu-Arabic numerals, and the development of modern algebraic notation. The last part is a collection of reflections on such topics as why notation is important, what makes good notation, and what physiological processes occur in the brain when we perceive mathematical symbols. This third section contains some of the most thought-provoking material in the book. Mazur has broken free from the shackles of historical narrative (which he seems to have been itching to do all along), and we roam with him as he meditates on deep questions about our mental processes. For instance, in "The Good Symbol," he examines how good notation can suggest new mathematical structures rather than merely represent known ones. In "Invisible Gorillas," the most ambitious chapter of the book, he surveys contemporary cognitive-psychological research on how the brain responds to perceptions of words and symbols. In the chapter titled "Conclusion," he makes the provocative move of likening symbols in poetry to those in mathematics.

In "Numerals," we find the history of numerical representation, from crude tallying schemes to positional notation requiring the use of 0 as a number. To illustrate the simultaneous fascination and puzzlement I experienced in reading this section, and indeed the whole book, consider p. 57. Just after the interesting observation that the expense of paper prevented the modern system of numerical representation

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from spreading more quickly (older systems allowed for computations that could be done on an abacus or sand table), there is a new subsection, one of only a handful in the book, given the brevity of the chapters. Already an odd organizational gambit, the new subsection begins with a very brief paragraph explaining why Pythagoras appears in a certain illustration. Immediately following, and seemingly unrelated, we have this statement: "Caliph stories provide the backdrops for so many anecdotal yarns that we sometimes forget that they are mostly the Western folk myths of an exotic bygone civilization." For a reader such as I, who was at the moment of reading discovering the existence of caliph stories, this is a puzzling way to introduce a new section. Indeed, only on the following page does one discover that the purpose of this subsection, which is less than three pages long, is to provide some account of how mathematical ideas passed from the Greeks to the Indians to the Arab world. I also found the chronology of "Numerals" particularly difficult to follow. To be fair, knowledge of such long-ago happenings is fragmentary, and the history itself is highly nonlinear. There is also an effort made to address the chronological thicket by including a foldout timeline at the end of "Numerals." But the appearance, for instance, of two different illustrations of the morphological development of modern numerals (pp. 37 and 78), containing overlapping dates but completely different content, does little to clarify what happened when.

In "Algebra," Mazur takes us from the algebraic notation of Euclid and Diophantus to that of Leibniz and Newton. Although still subject to the pleasures and frustrations of Mazur's informal style, I generally found this section more enlightening and compelling than the previous one. It contains, for instance, a detailed description of the work of one of history's earliest known algebraic innovators, Diophantus of Alexandria; an insightful essay on Descartes and his breakthrough linking of geometry and algebra; a chapter in praise of the notational acumen of Leibniz, many of whose innovations persist to the present; and, immediately following, a chapter on Newton's use and representation of the infinitesimal.

Mazur's writing is best described as conversational, with both the positive and negative implications of that word in evidence. Reading the book feels like having a long chat with a very knowledgeable friend who tells episodic stories, dispensing fascinating factual nuggets from his impressive store of reading, and peppering them with his own meditations. But his stories sometimes feel disjointed and meandering, and he is prone to the occasional overindulgence. There can be no doubt that this conversational quality enhances the book's readability, and substantially broadens its appeal beyond that of a scholarly text. Mazur's freewheeling style sometimes provides eruptions of beauty; witness the passage "We might think that [Diophantus's] notation must have hindered clear algebraic thinking. Perhaps, but routine and familiarity are the tailwinds of conception" (p. 108). It also sometimes yields cringeworthy moments, as when we are reminded that "there are no tweets telling us what went on in the minds of early contributors to mathematics" (p. 80).

Finally, I would be remiss not to mention the book's quite marvelous cover illustration, by Marcella Engel Roberts. It suggests a great wheel of life, decorated with diminishing concentric circles of symbols, some familiar but most ancient and mysterious. It provides yet another reason to buy a copy of Mazur's insightful, delightful, and frustrating book: each rediscovery of the cover image from under a pile of papers or tucked away on a shelf will give you the same frisson, the same sudden sense of the oneness of math with the great human effort to comprehend the world.

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