Math 236, Review handout for in-class exam

The exam will cover everything we've done in the course through Friday, February 1. It will be closed book and closed notes, and you'll have the full 60 minutes of class to do it. There will be approximately 6 problems, which will resemble homework problems in their phrasing; see the end of this handout for some practice problems. If you remember the relevant definitions and theorems, there won't be any great tricks required to do the problems (this doesn't mean they will all be easy, though!). One of the problems will be taken directly from the homework. You may use without proof any result given in class or in the book.

Some suggestions for reviewing: Do the practice exam on the following page (solutions will be available before the exam), study the definitions and theorems given below, work examples of each of the proof techniques listed below, and go over homework problems.

You should know the definitions of all of the following:

1. When two logical propositions are logically equivalent (p. 22)
2. Tautology and contradiction (p. 26)
3. Set equality, set containment (p. 33)
4. Union, intersection, set difference, set complement, disjoint sets (pp. 37-39)
5. The power set of a set (p. 106)
6. Cartesian product (p. 111)
7. Relation on a set A. Also reflexive relation, symmetric relation, anti-symmetric relation, transitive relation, partial ordering, and equivalence relation.

Some techniques you should be able to do well:

1. Make truth tables to determine logical equivalence
2. Write English statements in terms of variables and quantifiers, and then find the negations of these statements
3. Direct proof
4. Indirect proof (proof by contraposition, proof by contradiction)
5. Proof by induction (both usual induction and strong induction)
6. Prove if and only if statements

Some theorems you should know how to state and use:

1. Theorem 2 on p. 35: If A and B are sets, then $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$.
2. Theorem 1 on p. 73: The Division algorithm.
3. Theorem 4 on p. 107: If A is a set with exactly n elements, then $\mathcal{P}(A)$ has exactly 2^n elements.

Practice exam:

1. Suppose that P, Q and R are logical propositions. Determine whether $P \Rightarrow (Q \vee R)$ is logically equivalent to $(P \Rightarrow Q) \vee (P \Rightarrow R)$.

2. Find the negation of the following statement: For all positive integers n, there exists a positive integer k such that k is prime and $k^2 \leq n$.

3. Prove that for all $n \geq 1$,

$$\frac{1}{1 \cdot 5} + \frac{1}{5 \cdot 9} + \cdots + \frac{1}{(4n-3)(4n+1)} = \frac{n}{4n+1}.$$

4. Prove or disprove the following statements:

   (a) For all sets A and B, $\mathcal{P}(A \cup B) \subseteq \mathcal{P}(A) \cup \mathcal{P}(B)$.

   (b) For all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

   (c) For all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$.

5. Prove that $\log_3(5)$ is irrational. (Hint: recall that $\log_a(b) = c$ means $a^c = b$. You may also wish to use the fact that no number can be both divisible by 3 and not divisible by 3.)

6. Let $A = \{1, 2, 3, 4\}$, and consider the relation on A given by $R = \{(2,1), (1,3), (3,1), (3,3), (4,1)\}$. True or False:

   (a) Is R symmetric? Is R anti-symmetric?

   (b) Find the smallest relation S that contains R and is also transitive.

7. Let U be a set, and define a relation on the power set $\mathcal{P}(U)$ by setting $A \; R \; B$ if

$$(A - B) \cup (B - A) = \emptyset.$$

   Determine whether R is reflexive, symmetric, anti-symmetric, transitive.