Math 236 – Mathematicians of the week

Week 1: *Thales of Miletus (c. 624-547 BCE).* This was a long time ago. A very, very long time ago. Long enough that nothing he wrote survives. And nothing survives of the writing of anyone who ever saw any of Thales' writing. He's basically a figure of tradition, and ancient Greek tradition tended to attribute many discoveries to a few famous figures, regardless of their actual role in the discovery. But what is attributed to him is quite profound.

According to Aristotle, "Thales of Miletus taught that all things are water." As MacTutor explains, Thales believed that the Earth floats on water and all things come to be from water. For him the Earth was a flat disc floating on an infinite ocean. So Earthquakes were not fits of the gods, but rather waves on the water that led to the shaking of the Earth.

This may seem way out there – it did to me when I first encountered it an ancient philosophy course. In point of fact, it's not so far from our modern geological theories where some layers of the Earth are liquid. But the importance of Thales' views lies not in the precise nature of his claims, but rather that **he is the first recorded person who tried to explain natural phenomena by rational rather than by supernatural means.** As Sambursky writes, "It was Thales who first conceived the principle of explaining the multitude of phenomena by a small number of hypotheses for all the various manifestations of matter." He was the first one to look for first principles to explain things. This paved the way for deductive reasoning – given some first principles, you have to have some way to say what their consequences are (and with luck they include things you observe in nature, or otherwise believe to be true).

Thales is also credited with doing some math. The theorem called Thales' theorem says that a triangle inscribed in a semi-circle, with one side being the diameter of the semi-circle, must be a right triangle. As Wikipedia says, "He is the first known individual to use deductive reasoning applied to geometry, by deriving four corollaries to Thales' theorem."

Because of his role as originator of a pseudo-axiomatic mindset, and as one of the first recorded users of deduction reasoning, he's the first mathematician of the week in this class – whose focus will be on how to construct arguments about mathematical objects using deductive reasoning.

Week 2: *Euclid of Alexandria (c. 325-265 BCE).* Euclid lived in Alexandria, which is in modern-day Egypt. Like Thales, little is known about his life, and what comes down to us cannot really be substantiated. He is thought to have built up an active school of mathematics in Alexandria, and other mathematicians working there may have co-authored works that have been attributed only to Euclid.

Euclid is credited with writing *The Elements*, which is undisputedly the most influential mathematics book of all time, and quite possibly the most influential textbook of all time. Often in publishing, it is a mark of success to have new editions published; *The Elements* has had more editions (over a thousand) than any other book besides the Bible. What does it do? It is the first book to explicitly lay out the foundations of an intellectual discipline (in the form of a small set of "intuitively obvious" axioms), and then with careful and consistent logic, enumerate their consequences, culminating in deep results. For instance, two of Euclid's axioms are "A straight line segment can be drawn joining any two points" and "all right angles are congruent." From these, through logic and ingenuity, he deduces results such as a construction of a regular 15-gon, and even number theoretic results like the infinitude of the primes (Euclid thought of numbers in a geometric way, quite possibly because the Greek system of writing numbers was terrible).

While Thales may have been the first to try to explain diverse phenomena as emanating from first principles, Euclid took this mode of thought to a new level of clarity and consistency, and provided a template that has inspired thinkers for over 2000 years. I think of Euclid as "the great axiomatizer," although it might also be appropriate to add "and great deducer". The use of logic as the mode of mathematical argument is due in large part to him.

To illustrate the influence of *The Elements*, I can't do better than Wikipedia:

> The austere beauty of Euclidean geometry has been seen by many in western culture as a glimpse of an otherworldly system of perfection and certainty. Abraham Lincoln kept a copy of Euclid in his saddlebag, and studied it late at night by lamplight; he related that he said to himself, "You never can make a lawyer if you do not understand what demonstrate means; and I left my situation in Springfield, went home to my father's house, and stayed there till I could give any proposition in the six books of Euclid at sight".[14] Edna St. Vincent Millay wrote in her sonnet "Euclid alone has looked on Beauty bare", "O blinding hour, O holy, terrible day, When first the shaft into his vision shone Of light anatomized!". Einstein recalled a copy of the Elements and a magnetic compass as two gifts that had a great influence on him as a boy, referring to the Euclid as the "holy little geometry book".[15][16]

It's worth dwelling on the poem by Edna St. Vincent Millay (1892–1950):

Euclid alone has looked on Beauty bare

Euclid alone has looked on Beauty bare.
Let all who prate of Beauty hold their peace,
And lay them prone upon the earth and cease
To ponder on themselves, the while they stare
At nothing, intricately drawn nowhere
In shapes of shifting lineage; let geese
Gabble and hiss, but heroes seek release
From dusty bondage into luminous air.
O blinding hour, O holy, terrible day,
When first the shaft into his vision shone
Of light anatomized! Euclid alone
Has looked on Beauty bare. Fortunate they
Who, though once only and then but far away,
Have heard her massive sandal set on stone.

You may be surprised by Millay's view of mathematics – and particularly the axiomatic and deductive mathematics of *The Elements* – as the most direct access to the essence of beauty. But she is hardly alone. Most mathematicians have an aesthetic appreciation for the subject, and aesthetic terms like "deep" and "beautiful" are among the highest pieces of praise mathematicians give to a theorem. Here's Bertrand Russel on the subject:

> Mathematics, rightly viewed, possesses not only truth, but supreme beautya beauty cold and austere, like that of sculpture, without appeal to any part of our weaker

> nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight... is to be found in mathematics as surely as in poetry.

Week 3: *Abu Ja'far Muhammad ibn Musa al-Khwarizmi (c. 780-850).*

Perhaps the greatest mathematician of the Islamic-Arabic golden age (750 - 1258). During this time the state of the art of mathematics was maintained and expanded by the Persians, who established the incredible library, translation center, and research institute known as the House of Wisdom in Baghdad. Al-Khwarizmi was a resident scholar at the House of Wisdom, and his writings had a huge influence on the development of mathematics. His most famous work is on solving linear and quadratic equations. Its title, *al-Kitab al-mukhtasar fi hisab al-jabr wa'l-muqabala* roughly translates to *The compendious book on calculation by completion and balancing* (how come modern texts never have titles containing "compendious"?). The word *al-jabr* refers to the process of subtracting a term from one side of an equation and moving it to the other side, while *al-muqabala* refers to subtracting equal quantities from both sides of an equation. So they're very similar operations! And yet the word *al-jabr* became *algebra* in the west, giving us the name for the entire branch of math that studies solutions to equations. It could just as easily have been a variant of *al-muqabala.* Al-Khwarizmi was also aware of the positional decimal system for representing numbers, having read Indian texts that introduced the idea of zero. He wrote a book whose title was translated to *Al-Khwarizmi on the Hindu art of reckoning*, which helped to spread positional notation throughout the Middle East and Europe. His influence was such that his name itself became the word *algorithm.*

While his text on algebra was fundamental, it bore little resemblance to modern texts. For one thing, our present mathematical notation was almost completely absent. Al-Khwarizmi's text was entirely in prose, as in this snippet:

> If the instance be, 'ten and thing to be multiplied by thing less ten,' then this is the same as 'if it were said thing and ten by thing less ten.'

In modern notation, this is simply $(10+x)(x-10) = (x+10)(x-10)$. Sure makes you appreciate our modern way of writing things! Somehow the insight of Diophantus – who used a symbol to denote the unknown quantity – has not made it to Al-Khwarizmi, who instead uses "thing." This illustrates how the history of math is not a linear progression; good ideas are discovered and lost, and then discovered again. (Sources: http://ualr.edu/lasmoller/aljabr.html, Wikipedia).

Week 4: *Leonardo di Pisa (Fibonacci) (c. 1170-1250).* The son of a wealthy Italian merchant, Leonardo lived most of his early life in Bugia, a port city in what is now Algeria, where his father had a trading post. Given an education by the Moors, he was exposed to the Hindu-Arabic positional system of numeration, which was prevalent in the Middle East and Northern Africa thanks to the influence of Al-Khwarizmi and other Middle Eastern mathematicians. Leonardo recognized the superiority of this system to Roman numerals, which were still the standard in Europe. Think about it for a minute: how do you multiply the numbers represented by the Roman numerals XVII and XIV? Probably you convert them to the Hindu-Arabic 17 and 14 in your head, and then do it. Fibonacci's great contribution was his book *Liber Abaci* (written in Latin), whose title translates to "Book of calculation." In it, he laid out example after example showing the the advantages of the Hindu-Arabic system over Roman numerals: in currency exchange, bookkeeping, engineering, etc.

In *Liber Abaci*, he also described other mathematical problems that caught his interest. One such problem involved some rabbits: say that you begin with one pair of adult rabbits (denote this by A). Each month, every adult pair has a pair of offspring (denote this by K, for kids), and any kids become adults. Over the first few months, your rabbit population looks like this: A, AK, AKA, AKAAK, AKAAKAKA, AKAAKAKAAKAAK, so that the number of pairs you have is 1, 2, 3, 5, 8, 13. Indeed, representing the number of pairs at time n by F_n, it's not hard to discern the relation $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$, since F_{n-1} gives the number of adult pairs at time n, and F_{n-2} gives the number of kid pairs. Leonardo mentioned this problem only as an example, and didn't study the sequence in much depth. Nor was he the first to examine it: it had been known for centuries to Indian mathematicians. Yet today this sequence bears Fibonacci's name, and is among the most heavily studied objects in mathematics (there is a journal called the *Fibonacci quarterly* devoted exclusively to papers relating to it, and many famous problems involving the sequence remain unsolved). In the popular historical memory, the little matter of *Liber Abaci* revolutionizing the way the West represents numbers has been thoroughly overshadowed by one of its examples, mentioned in passing.

Week 5: **Pierre de Fermat (1601 or 1607/08 - 1665)**. Fermat was a lawyer at the Parlement de Toulouse, where he was known for his long, wispy hair. OK, so I made that up, but he really did have long, shaggy locks that grayed as he grew older (check out some of the portraits of him online), and his hair was not thick. This was likely just the style of the times for people of lofty social standing.

Fermat was not a professional mathematician – indeed university posts were rare at the time – but rather an amateur, and likely the most famous and influential amateur mathematician of all time. His status as a successful professional who did math on the side suited him quite well, as we much preferred conjecturing and sketching to rigorously writing out proofs. He transmitted most of his work in the form of letters to fellow mathematicians and famously in notes scrawled in the margins of his copy of Diophantus' *Arithmetica*, which inspired him to great flights of mathematical imagination.

Fermat made fundamental contributions to a wide variety of mathematical disciplines. He helped lay the foundations of analytic geometry (think Cartesian coordinates) and thus paved the way for calculus. He rigorously resolved a question from a professional gambler (who wondered why he won in the long run when he bet on rolling at least one six in four throws of a die, but lost in the long run when he bet on rolling at least one double-six in 24 throws of two dice). In so doing, he performed the first probability theory calculation, and is now considered one of the founders of probability theory. He made contributions to optics as well. And of course he was a hugely important number theorist, probably standing along with Euclid among those coming before Euler. He studied perfect numbers extensively, and during the course of those investigations he discovered his little theorem. He made giant contributions to the understanding of Diophantine equations, primes, and essentially all the other core topics of number theory.

He also stands apart for his incredible intuition and ability to ask outstanding questions. It's hard to overstate the importance of asking good questions, in mathematics as well as in the rest of life. Fermat had a knack for articulating questions that were plausible but hard to definitively settle. Most famous among his questions is what became known as Fermat's Last Theorem: the assertion that for a fixed integer $n \geq 3$, the Diophantine equation $x^n + y^n = z^n$ is only solvable in integers (x, y, z) if one of those integers is zero (so-called trivial solutions). It's an audacious claim. Fermat scribbled in the margin of Arithmetica that he had "discovered a marvelous proof of this theorem, which this margin is too small to contain" (except he scribbled it in Latin, of course). But no such proof was ever found, and it seems unlikely that Fermat had a correct proof. His assertion was finally proved in 1995 by Andrew Wiles, who used methods building on a huge modern theory of numbers, which Fermat had no knowledge of.

A shadow side of Fermat's intuitive approach to math and asking questions comes in his theory about Fermat numbers. These are numbers of the form $2^{2^n} + 1$, so that the first few are $3, 5, 17, 257, 65537$. Fermat knew these, and he noted that all of them were prime. He conjectured that the rest were prime, too. But Euler showed the next Fermat number $2^{2^{32}} + 1$ is composite. And indeed the next twenty have now been showed to be composite as well. The conjecture is now that the *only* prime Fermat numbers are the ones Fermat knew about. That's a fairly spectacularly failed conjecture.

Week 6: **Sophie Germain (1776 - 1831)**. Born in Paris, to a wealthy family (father was probably a silk merchant). She was able to rely on family financial support for her entire adult life. Instead of getting into gambling, as other aristocrats of the time did, she got into books. Her family had a ton of them, as her father was a political figure, and aristocrats of the time tended to maintain significant libraries anyway.

When Germain was 13, the Bastille fell, and the revolutionary atmosphere of the city forced her to stay inside. For entertainment she turned to her father's library. Here she found J. E. Montucla's L'Histoire des Mathmatiques, and his story of the death of Archimedes intrigued her.

Sophie Germain thought that if the geometry method, which at that time referred to all of pure mathematics, could hold such fascination for Archimedes, it was a subject worthy of study. So she pored over every book on mathematics in her father's library, even teaching herself Latin and Greek so she could read works like those of Sir Isaac Newton and Leonhard Euler.

Germain's parents did not at all approve of her sudden fascination with mathematics, which was then thought inappropriate for a woman. When night came, they would deny her warm clothes and a fire for her bedroom to try to keep her from studying, but after they left she would take out candles, wrap herself in quilts and do mathematics. As Lynn Osen describes, when her parents found Sophie "asleep at her desk in the morning, the ink frozen in the ink horn and her slate covered with calculations," they realized that their daughter was serious and relented. After some time, her mother even secretly supported her.

In 1794, when Germain was 18, the Ecole Polytechnique opened. As a woman, Germain was barred from attending, but the new system of education made the "lecture notes available to all who asked." The new method also required the students to "submit written observations." Germain obtained the lecture notes and began sending her work to Joseph Louis Lagrange, a faculty member. She used the name of a former student Monsieur Antoine-August Le Blanc, "fearing," as she later explained to Gauss, "the ridicule attached to a female scientist." When Lagrange saw the intelligence of M. LeBlanc, he requested a meeting, and thus Sophie was forced to disclose her true identity. Fortunately, Lagrange did not mind that Germain was a woman, and he became her mentor. He visited her in her home, giving her moral support.

In 1808, the German physicist Ernst F F Chladni had visited Paris where he had conducted experiments on vibrating plates, exhibiting the so-called Chladni figures. The Institut de France set a prize competition with the following challenge:

formulate a mathematical theory of elastic surfaces and indicate how it agrees with empirical evidence.

A deadline of two years for all entries was set.

Most mathematicians did not attempt to solve the problem, because Lagrange had said that the mathematical methods available were inadequate to solve it. Germain, however, spent the next decade attempting to derive a theory of elasticity, competing and collaborating with some of the most eminent mathematicians and physicists. In fact, Germain was the only entrant in the contest in 1811, but her work did not win the award. She had not derived her hypothesis from principles of physics, nor could she have done so at the time because she had not had training in analysis and the calculus of variations. Her work did spark new insights, however. Lagrange, who was one of the judges in the contest, corrected the errors in Germain's calculations and came up with an equation that he believed might describe Chladni's patterns.

The contest deadline was extended by two years, and again Germain submitted the only entry. She demonstrated that Lagrange's equation did yield Chladni's patterns in several cases, but could not give a satisfactory derivation of Lagrange's equation from physical principles. For this work she received an honourable mention.

Germain's third attempt in the re-opened contest of 1815 was deemed worthy of the prize of a medal of one kilogram of gold, although deficiencies in its mathematical rigour remained. To public disappointment, she did not appear as anticipated at the award ceremony. Though this was the high point in her scientific career, it has been suggested that

Fermat's Last Theorem can be divided into two cases. Case 1 involves all p that do not divide any of x, y, or z. Case 2 includes all p that divide at least one of x, y, or z. Germain proposed the following, commonly called "Sophie Germain's theorem":

Theorem 1 *Let p be an odd prime number such that $2p+1$ is also prime. Then the first case of Fermat's Last Theorem holds true for p. That is, there are no positive integers x, y, z with $x^p + y^p = z^p$ and $p \nmid xyz$*

To this day, prime numbers p such that $2p + 1$ is also prime are called *Sophie Germain primes.*

Here are Gauss' words about Germain, written upon learning her true identity in 1806:

"How can I describe my astonishment and admiration on seeing my esteemed correspondent M. leBlanc metamorphosed into this celebrated person... when a woman, because of her sex, our customs and prejudices, encounters infinitely more obstacles than men in familiarising herself with [number theory's] knotty problems, yet overcomes these fetters and penetrates that which is most hidden, she doubtless has the most noble courage, extraordinary talent, and superior genius."

Week 7: **Carl Friederich Gauss (1777-1855)** You all know at least one of Gauss' many mathematical discoveries – the Gaussian distribution (a.k.a Bell Curve). But there are indeed many.

Gauss was a mathematical prodigy from an early age. A famous story has it that in primary school after the young Gauss misbehaved, his teacher, J.G. Buttner, gave him a task: add a list of integers in arithmetic progression; as the story is most often told, these were the numbers from 1 to 100. The young Gauss reputedly produced the correct answer within seconds, to the astonishment of his teacher and his assistant Martin Bartels.

Gauss's presumed method was to realize that pairwise addition of terms from opposite ends of the list yielded identical intermediate sums: $1 + 100 = 101, 2 + 99 = 101, 3 + 98 = 101$, and so on, for a total sum of $50 \times 101 = 5050$. However, the details of the story are at best uncertain; some authors, such as Joseph Rotman in his book A first course in Abstract Algebra, question whether it ever happened.

In 1788 Gauss began his education at the Gymnasium with the help of Buttner and Bartels, where he learnt High German and Latin. After receiving a stipend from the Duke of Brunswick-Wolfenbuttel [editor's note: wait, what? How does an 15-year-old get a stipend from a Duke to do math?], Gauss entered Brunswick Collegium Carolinum in 1792. At the academy Gauss independently discovered Bode's law, the binomial theorem and the arithmetic- geometric mean, as well as the law of quadratic reciprocity and the prime number theorem. [editor's note: OK, so this partially explains the answer to the last question: a 15-year-old who can discover all of this fundamental stuff.]

In 1795 Gauss left Brunswick to study at Gottingen University. Gauss left Gottingen in 1798 without a diploma, but by this time he had made one of his most important discoveries - the construction of a regular 17-gon by ruler and compasses This was the most major advance in this field since the time of Greek mathematics and was published as Section VII of Gauss's famous work, Disquisitiones Arithmeticae, published in 1801.

In June 1801, Zach, an astronomer whom Gauss had come to know two or three years previously, published the orbital positions of Ceres, a new "small planet" which was discovered by G Piazzi, an Italian astronomer on 1 January, 1801. Unfortunately, Piazzi had only been able to observe 9 degrees of its orbit before it disappeared behind the Sun. Zach published several predictions of its position, including one by Gauss which differed greatly from the others. When Ceres was rediscovered by Zach on 7 December 1801 it was almost exactly where Gauss had predicted. Although he did not disclose his methods at the time, Gauss had used his least squares approximation method.

As a final illustration of Gauss' mathematical prowess, there is a famous statement in number theory called the law of quadratic reciprocity. Before Gauss, it was a conjecture; no one had ever given a complete proof. By the time he was 30, Gauss had given six different proofs of quadratic reciprocity, and he would go on to give at least two more (see https://www.rzuser.uni-heidelberg.de/hb3/rchrono.html).

Week 8: **Georg Cantor (1845-1918):** Cantor was born in St. Petersburg, and always felt attached to Russia, though he lived his entire life from age 11 in Germany. His father wanted him to be an engineer, but reluctantly consented to let Cantor study math, to Cantor's great joy. Cantor wrote his dissertation on number theory, but then turned to analysis when the famous analyst Heine (who was on the faculty with Cantor at Halle) challenged him to prove a famous conjecture in Fourier analysis that had resisted the efforts of many brilliant mathematicians. Amazingly, Cantor did just that, proving the theorem in 1870. For the next few years his attention turned to analysis. In the course of his research in analysis, he became interested in subsets of the real numbers, and his interest evolved into set theory. He went back to the very foundations, notably the questions we've addressed in class: how can one tell the size of a set? By 1874 he had made staggering progress in this area, showing that the rational numbers are countable, the algebraic numbers are countable, and the real numbers are uncountable. Within another year he had shown that the closed interval $[0, 1]$ of real numbers is equinumerous to $\mathbb{R}^n$ for any $n \geq 1$. He wrote of this result: "I see it, but I don't believe it!"

By this time Cantor's ideas about infinite sets had aroused considerable opposition in some quarters of the mathematical world. This opposition was on philosophical grounds – critics rejected the nature of Cantor's methods. Leopold Kronecker was especially outspoken. He was a constructionist, and rejected any existence proof that did not involve a construction of the object in question. He also rejected proof by contradiction. He had an animosity towards the idea of "actual infinity," namely that we can reason about infinity in a sensical way; he felt that infinity was an abstraction that was not mathematically legitimate (a view supported by some religious scholars, incidentally – infinity was supposed to be the sole province of God. Cantor, a devout Lutheran, felt that was about "ultimate infinity," but not about the smaller versions covered in his theory). From his perch as a well-known mathematician, Kronecker launched attacks not only on Cantor's ideas but on Cantor himself, calling him a "scientific charlatan," "renegade," and "corrupter of youth." Henri Poincare also opposed Cantor's ideas. He was an intuitionist, and believed that mathematical entities cannot be reduced to logical propositions, originating instead in the intuitions of the mind. Moreover, the human mind cannot intuitively construct an infinite set, and so intuitionists disallow the idea of infinity as an expression of any sort of reality. Kronecker articulated this view somewhat with his quotation, "God made the integers. All the rest is the work of man." Poincare called Cantor's ideas "a grave disease" infecting the discipline of mathematics. This fierce public opposition weighed on Cantor. He suffered recurring bouts of depression throughout the last 35 years of his life, and had several periods in sanitoria. The latter part of his life saw him become consumed with the theory that Francis Bacon wrote the works of Shakespeare (a theory rejected by all present-day mainstream Shakespeare scholars). However, it should be noted that Cantor was almost certainly predisposed to mental illness, and it is likely that his depression was not caused by this professional adversity.

Cantor also had many defenders, whose number grew over time. Dedekind corresponded fruitfully with Cantor (who helped give him the idea for his "Dedekind cut" construction of the real numbers) early in Cantor's career, and helped him get papers published. In 1926, eight years after Cantor's death, the great David Hilbert mounted a ringing defense of him: "From the paradise that Cantor created for us, no one can expel us." He meant that the theorems made possible by Cantor's ideas – which are many and continue to this day – were his gift to us. Hilbert described Cantor's work as "...the finest product of mathematical genius, and one of the supreme achievements of purely intellectual human activity." Today the fact that Cantor's theory is a standard part of the undergraduate mathematics curriculum testifies to its near-universal acceptance.

Week 9: **Emmy Noether (1882-1935):** (Note: the following is taken from the outstanding entry on Noether from Wikipedia, as well as the entry on Noether at https://www.sdsc.edu/ScienceWomen/.)

Amalie Emmy Noether spent an average childhood learning the arts that were expected of upper middle class girls. Girls were not allowed to attend the college preparatory schools. Instead, she went to a general "finishing school," and in 1900 was certified to teach English and French. But rather than teaching, she pursued a university education in mathematics

Noether showed early proficiency in French and English. In the spring of 1900, she took the examination for teachers of these languages and received an overall score of sehr gut (very good). Her performance qualified her to teach languages at schools reserved for girls, but she chose instead to continue her studies at the University of Erlangen.

This was an unconventional decision; two years earlier, the Academic Senate of the university had declared that allowing mixed-sex education would "overthrow all academic order".[17] One of only two women in a university of 986 students, Noether was only allowed to audit classes rather than participate fully, and required the permission of individual professors whose lectures she wished to attend. Despite these obstacles, on 14 July 1903 she passed the graduation exam at a Realgymnasium in Nuremberg.

Soon after arriving at Gottingen, however, she demonstrated her capabilities by proving the theorem now known as Noether's theorem, which shows that a conservation law is associated with any differentiable symmetry of a physical system. The paper was presented by a colleague, F. Klein on 26 July 1918 to a meeting of the Royal Society of Sciences at Gottingen. Noether presumably did not present it herself because she was not a member of the society. American physicists Leon M. Lederman and Christopher T. Hill argue in their book Symmetry and the Beautiful Universe that Noether's theorem is "certainly one of the most important mathematical theorems ever proved in guiding the development of modern physics, possibly on a par with the Pythagorean theorem".

Yet she is probably best-remembered for her work in abstract algebra. In his introduction to Noether's Collected Papers, Nathan Jacobson wrote that "the development of abstract algebra, which is one of the most distinctive innovations of twentieth century mathematics, is largely due to her – in published papers, in lectures, and in personal influence on her contemporaries." She sometimes allowed her colleagues and students to receive credit for her ideas, helping them develop their careers at the expense of her own.

She was the first to introduce the definition of a commutative ring – which you'll see if you take abstract algebra! She proved such deep theorems about these structures that a very broad class of commutative rings are now known as "Noetherian rings".

She labored for years in Germany as basically an adjunct professor, with no title and paid almost nothing. This for one of the great mathematical geniuses of the 20th century. Despite this lack of official recognition, she was widely known as a great mathematician among her colleagues, and eventually won some major professional honors (invited speaker at the 1932 ICM, a major professional prize). She is now universally regarded as one of the greatest modern mathematicians.

Noether's mathematical work has been divided into three "epochs". In the first (1908-1919), she made contributions to the theories of algebraic invariants and number fields. Her work on differential invariants in the calculus of variations, Noether's theorem, has been called "one of the most important mathematical theorems ever proved in guiding the development of modern physics". In the second epoch (1920-1926), she began work that "changed the face of [abstract] algebra". In her classic 1921 paper Idealtheorie in Ringbereichen (Theory of Ideals in Ring Domains) Noether developed the theory of ideals in commutative rings into a tool with wide-ranging applications. She made elegant use of the ascending chain condition, and objects satisfying it are named Noetherian in her honor. In the third epoch (1927-1935), she published works on noncommutative algebras and hypercomplex numbers and united the representation theory of groups with the theory of modules and ideals. In addition to her own publications, Noether was generous with her ideas and is credited with several lines of research published by other mathematicians, even in fields far removed from her main work, such as algebraic topology.