Carleton College, Winter 2013
Math 232, Practice Midterm 2
Prof. Jones

Note: the exam will have 3-4 questions like the ones below, as well as a section of true/false questions similar to the one on the first exam. To review for those, the best thing is to work the review questions listed on the main course webpage.

1. Find a bases for the kernel and image of the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ -3 & 0 & 0 & 1 & 0 \\ -1 & 0 & 4 & 1 & 2 \end{bmatrix}.$$

2. Find a basis for the following subspace of P_4.

$$W = \{p(x) \in P_4 \mid p(1) = p(-1) = 0\}.$$

What is the dimension of W?

3. Determine whether the following mappings are linear transformations. Either prove that a given map is linear or give a counterexample to show it's not linear.

(a) $T : \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T((x_1, x_2)) = (2x_1, x_1 + 4, 5x_2)$

(b) $T : P_2 \to P_3$ defined by $T(a_2x^2 + a_1x + a_0) = a_0x^3 + (a_1 - a_0)x^2 + 3a_2 - (1/2)a_0$

4. Let V be a finite-dimensional vector space, and let S be a linearly independent subset of V. Let S' be a proper subset of S (this means that $S' \subseteq S$ and $S' \neq S$). Prove that S' cannot be a basis for V. [Hint: use Theorem 3.3.4.]

5. (a) Consider the mapping $T : \mathbb{R}^{2\times 2} \to \mathbb{R}^{2\times 2}$ defined by

$$T\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a+b & c-b \\ b+2d-3c & d+4a \end{pmatrix}.$$

Prove that T is a linear transformation.

(b) Given the basis $\alpha = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \right\}$ of $\mathbb{R}^{2\times 2}$,

give the matrix $[T]_\alpha$ of T with respect to the basis α.

6. The mapping $T : \mathbb{R}^2 \to P_2$ given by $T((a_1, a_2)) = (a_1 + a_2)x^2 + a_2x + a_1$ is a linear transformation.

(a) Prove that $\alpha = \{(1, 2), (-1, 0)\}$ is a basis for $\mathbb{R}^2$ and $\beta = \{x^2 + 2, x^2 + x, 1\}$ is a basis for P_2.

(b) Find the matrix $[T]_\alpha^\beta$

(c) What is the dimension of Ker(T)? Find a basis for Ker(T).

(d) What is the dimension of Im(T)? Find a basis for Im(T)