Carleton College, Winter 2014
Math 121, Practice Midterm 2
Prof. Jones

Note: the exam will have a section of true-false questions, like the one below.

1. True or False. Briefly explain your answer. An incorrectly justified answer may not receive full (or any) credit.

   (a) Suppose that as a car brakes, its deceleration is proportional to the square of its velocity. Then its motion is described by the differential equation

   $$\frac{dv}{dt} = \frac{k}{v^2},$$

   where $v(t)$ is the car's velocity at time t and k is a constant.

   (b) In part (a), the constant k is positive.

   (c) Suppose that for the series $\sum_{n=1}^{\infty} a_n$, the sequence of terms a_n satisfies $\lim_{n\to\infty} a_n = 0$. Then $\sum_{n=1}^{\infty} a_n$ might converge, or might diverge; there is not enough information to tell.

   (d) If $f(x)$ is a function with $f(0) = 0$, $f'(0) = 1$ and $f''(0) = -2$, then its second Taylor polynomial at $a = 0$ is $T_2(x) = x - 2x^2$.

   (e)Let $T_2(x)$ and $T_4(x)$ be the degree 2 and 4 MacLaurin polynomials for a function $f(x)$. Then $T_4(2)$ is always a better approximation to $f(2)$ than $T_2(2)$.

2. For $f(x) = xe^x$, find the fourth Taylor polynomial $T_4(x)$ at $a = 0$. Then use it to approximate $-\frac{1}{e}$.

3. Solve the initial value problem

   $$\frac{dy}{dx} = \frac{y}{x}, \qquad y(1) = 3$$

4. Solve the initial value problem

   $$\frac{dy}{dx} = \frac{y^2}{x^2+1}, \qquad y(0) = 1$$

5. Determine whether these series converge. If a series converges and is geometric, find its sum.

   a) $\displaystyle\sum_{n=1}^{\infty} (2 + (-1)^n)$

   b) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{\pi}{e^n}\right)$

   c) $\displaystyle\frac{8}{9} - \frac{16}{27} + \frac{32}{81} - \frac{64}{243} + \cdots$