Carleton College, Winter 2014
Math 121, Practice Final
Prof. Jones

Note: the exam will have a section of true-false questions, like the one below.

1. True or False. Briefly explain your answer. An incorrectly justified answer may not receive full (or any) credit.

   (a) It's possible for a power series to converge for all x in $(1, 2]$ and for $x = 0$ but not for any other value of x.

   (b) Denote by $g(x)$ the twentieth derivative of $f(x) = xe^{-x^4}$. Then $g(0) = 0$.

   (c) Suppose that for the series $\sum_{n=1}^{\infty} a_n$, the sequence of terms a_n satisfies $\lim_{n\to\infty} a_n = 0$. Then $\sum_{n=1}^{\infty} a_n$ might converge, or might diverge; there is not enough information to tell.

   (d) To evaluate $\int \frac{2x^2-2}{x^3-3x}\, dx$, you must use partial fractions.

   (e) The Taylor series of any function $f(x)$ at $x = 0$ converges for all values of x.

2. Find the following integrals:

   (a) $\int \ln x \, dx$

   (b) $\int_1^{\infty} \frac{\ln x}{x}\, dx$

   (c) $\int \frac{x^3}{\sqrt{1-x^2}}\, dx$

3. Solve the initial value problem

$$\frac{dy}{dx} = \frac{y^2}{x^2+1}, \qquad y(0) = 1$$

4. The integral $\int_0^1 e^{-x^3}\, dx$ cannot be evaluated exactly. Use a method from the course this term to approximate this integral *to within an error of at most 1/60*. Leave your answer as a fraction. (So select a method where finding the error bound won't be to difficult)

5. Describe another method for approximating the integral from the previous problem, and write down (but do not evaluate) a sum of five numbers that is an approximation of this integral.

6. Use ideas from the course to approximate e^2 to within 0.1. Leave you answer as a fraction, but explain why your approximations are correct to within 0.1.

7. Determine whether these series converge. If a series converges and is geometric, find its sum.

a) $\displaystyle\sum_{n=2}^{\infty} \frac{2^n}{n^2-1}$

b) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$ [Hint: it's helpful to write $(n+1)^{n+1}$ as $(n+1)^n(n+1)$.]

d) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n-\ln n}$

8. Determine whether this series converges absolutely, converges conditionally, or diverges:

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}.$$

9. Determine the interval of convergence of the power series $\displaystyle\sum_{n=1}^{\infty} \frac{(x+1)^n}{n^2 3^n}$.