Math 121 additional review problems for midterm 2

A1. Let $\sum_{n=1}^{\infty} a_n$ be a series. What do we mean by the sequence of terms of this series? What do we mean by the sequence of partial sums?

A2. Give the definition of convergence for a series $\sum_{n=1}^{\infty} a_n$.

A3. State the nth term test for divergence of a series $\sum_{n=1}^{\infty} a_n$.

A4. True or false. Briefly explain your answer.

(a) Let $T_{10}(x)$ be the nth Taylor polynomial for a function $f(x)$, centered at 5. If the eleventh derivative $f^{(11)}(x)$ has very large absolute value on the interval $[5, 7]$, then the error bound for the approximation of $f(7)$ by $T_{10}(7)$ will be large.

(b) A good translation of the sentence "A particle slows at a rate proportional to the square of the time it has been moving" into symbolic language is

$$\frac{dV}{dt} = t^2,$$

where V is the velocity of the particle at time t.

(c) A sequence can only converge when its terms approach zero as n approaches infinity.

(d) The sum of the series $\sum_{n=2}^{\infty} n \left(\frac{1}{3}\right)^n$ is

$$\frac{2/9}{1-(1/3)} = \frac{2/9}{2/3} = \frac{1}{3},$$

since $r = 1/3$ and the first term is $2/9$.

(e) The series $\sum_{n=0}^{\infty} \tan^{-1}(n)$ converges.

(f) Let a_n denote the sequence whose nth term is the nth digit of π after the decimal point, so $a_1 = 1$, $a_2 = 4$, $a_3 = 1$, $a_4 = 5$, $a_5 = 9$, and so on. What can you say about the series $\sum_{n=1}^{\infty} \frac{a_n}{10^n}$?

A5. You have an old car whose maximum speed is 60 mph. When you floor it, its acceleration is proportional to the difference between its maximum speed and its current speed. If this machine needs only one minute to accelerate from rest to a blazing 20 mph, how long will it take to reach 50 mph? (You should start this problem by writing down a differential equation, and then solving it).

A6. Use Simpson's rule with $n = 2$ to approximate $\tan^{-1}(1)$ (you'll need to rephrase this question using an integral in order to apply Simpson's rule). Then use a Taylor polynomial with $n = 3$ to approximate the same number. Then say which approximation has a better error bound (note that you do not need to memorize any error bound formulas for the exam. For this problem, it will be helpful to look them up in your notes or in the book.)

A7. Review your homework problems about Chloe! Solutions will be available on Monday.