Math 121: Solutions to selected review problems for the first exam

Ch. 7 Review Exercises #13: Begin by doing integration by parts, with $u = x^2$ and $dv = e^{4x}\,dx$. Then you'll need to do IBP again, with $u = x$ and $dv = e^{4x}\,dx$.

Ch. 7 Review Exercises #29: Convert two of the $\cos x$ terms to $\sin x$ via $\cos^2 x = (1 - \sin^2 x)$ to get

$$\int \sin^5 x(1 - \sin^2 x)\cos x.$$

Now substitute $u = \sin x$ to get a much easier integral.

Ch. 7 Review Exercises #35: This one is ripe for partial fractions, since it's the integral of a rational function whose denominator is already factored. Write

$$\frac{1}{(t-3)^2(t+4)} = \frac{A}{t-3} + \frac{B}{(t-3)^2} + \frac{C}{t+4},$$

and then multiply through by the common denominator $(t-3)^2(t+4)$ to get

$$1 = A(t-3)(t+4) + B(t+4) + C(t-3)^2.$$

We can find A, B, and C by equating coefficients, or we can set $t = 3$ to get $B = 1/7$, and set $t = -4$ to get $C = 1/49$. Then we can plug in $t = 1$ to give us $1 = -10A + 5/7 + 4/49$, which works out to $A = -1/49$. So our integral becomes

$$-\frac{1}{49}\int \frac{1}{t-3}\,dt + \frac{1}{7}\int \frac{1}{(t-3)^2}\,dt + \frac{1}{49}\int \frac{1}{t+4}\,dt$$

and this equals

$$-\frac{1}{49}\ln|t-3| - \frac{1}{7}\frac{1}{t-3} + \frac{1}{49}\ln|t+4| + C,$$

or

$$\frac{1}{49}\ln\left|\frac{t+4}{t-3}\right| - \frac{1}{7}\frac{1}{t-3} + C.$$

Ch. 7 Review Exercises #57: Start with integration by parts: let $u = \ln(x^2+9)$ and $dv = 1\,dx$. This gives

$$\int \ln(x^2+9)\,dx = x\ln(x^2+9) - 2\int \frac{x^2}{x^2+9}\,dx.$$

To do the integral on the right-hand side, use trig substitution: take $x = 3\tan\theta$. So

$$\int \frac{x^2}{x^2+9}\,dx = \int \frac{9\tan^2\theta}{9\sec^2\theta} 3\sec^2\theta\,d\theta = 3\int \tan^2\theta\,d\theta.$$

Now we can do something clever with the last integral: writing $\tan^2\theta = \sec^2\theta - 1$, it becomes $3\int \sec^2 - 1\ d\theta$, which equals $3\tan\theta - 3\theta + C$. But this is the same as $x - 3\tan^{-1}(x/3) + C$. So putting this all together, we get

$$\int \ln(x^2+9)\,dx = x\ln(x^2+9) - 2x + 6\tan^{-1}(x/3) + C.$$

Ch. 7 Review Exercises #79: This gives $\lim_{t\to-\infty}\left[\tan^{-1}(x)\right]_t^0$, which is

$$0 - \lim_{t\to-\infty} \tan^{-1}(t).$$

But recall from the graph of $y = \tan^{-1}(x)$ that there are horizontal asymptotes at $y = \pi/2$ (as $x \to \infty$) and $y = -\pi/2$ (as $x \to -\infty$). Thus our improper integral converges to $\pi/2$.

Ch. 7 Review Exercises #86: Note that since $-1 \leq \sin x \leq 1$, we have

$$0 \leq (\sin^2 x)e^{-x} \leq e^{-x}.$$

It is important to have $(\sin^2 x)$ instead of just $\sin x$, since we need the function to always be greater than or equal to 0. Now it's easy to show that $\int_8^\infty e^{-x}\,dx$ converges (we did this in class). So $\int_8^\infty (\sin^2 x)e^{-x}\,dx$ must also converge.