# Stern Perfection: Mathematics as a Fine Art

*By Rafe Jones*

> Mathematics, rightly viewed, possesses not only truth, but supreme beauty – a beauty cold and austere, like that of sculpture, without appeal to any part of our weaker nature, without the gorgeous trappings of painting or music, yet sublimely pure, and capable of a stern perfection such as only the greatest art can show. The true spirit of delight… is to be found in mathematics as surely as in poetry.
>
> – Bertrand Russell, logician and philosopher

Math and science are so firmly established in the public's perception as partner disciplines, one wonders if they are separate words any more. We test for aptitude in "math and science" as opposed to the "arts and humanities." We speak of math-science people. Math and science, in this culture, seem to go together. The question, however, of whether math *is* a science, is altogether different.

Pure mathematics – that without an eye towards application – has almost nothing in common with science. Not only does it have no connection to the physical world, but it is done solely for its own sake, and, I will argue, for the sheer beauty of it. Thus, a far better partner discipline for math is the fine arts.

Science is inseparable from the physical world it describes. Observation, the first pillar of the scientific method, binds scientists to the physical universe. Most crucial in the early stages of every science, observation helps people organize their perceptions of the world into patterns. More advanced science holds observation and theory in a delicate dance. For instance, Physicist Albert Einstein's theory of relativity gained acceptance on largely theoretical grounds well before its real-world predictions confirmed it.

Elementary mathematics, too, derives much inspiration from the real world. The concepts of lines, rectangles, circles, and other basic geometric figures are directly suggested by the physical world. Even in the famously esoteric field of number theory, the basic objects – the whole numbers – were first conceived to count physical objects.

However, more advanced mathematical concepts paradoxically lose all relation to the physical world from which they are derived. The intensely-scrutinized complex numbers are a single logical leap from the real numbers yet bear no analogy to our experience of real numbers in the "real" world. The same could be said of infinite series, linear operators, Cantor sets, and p-adic numbers. Laws governing the behavior of these objects are determined not by experiment, but by proof. Finding a proof, in turn, involves searching only one's imagination and resident mental library of mathematical results. It is no coincidence that mathematicians don't have labs or machines: their work demands no interrogation or special arrangement of the physical world. They can work anywhere, under virtually any circumstances.

Besides their differences regarding the physical world, math and science have very different assumptions about truth. Truth in science is messy and provisional. It is debated and defined by a consensus of scientists, men and women, and it changes over time. By contrast, truth in the mathematical universe is unambiguous. There are essentially never disputes over a statement's veracity. However, the price for this clear notion of truth is that the objects of mathematical assertions – even the ones inspired by physical objects – do not exist in the real world. The statement that the ratio of a circle's circumference to its diameter is *pi* is mathematically true. Yet every object resembling a circle in the physical world has tiny bumps and imperfections that prevent it from being an actual circle. Hence, to have any meaning in the physical world, the statement must be profoundly changed. Some contend the mathematical statement holds only in the universe of abstraction, but the nature of this universe and its relation to the physical one are far from obvious. One might say in math, truth is clear but the universe to which it applies is mysterious, whereas in science, the universe is clear but truth is mysterious.

Despite these deep differences, math has found many remarkable applications in science – and it is thanks to them that the two disciplines are so firmly associated. There seems to be no good reason that mathematical precepts formulated in a vacuum, so to speak, should be able to describe the physical world. This is precisely the thesis of a well-known article by Eugene Wigner entitled, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences." In it, the author declares, "The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a gift we neither understand nor deserve. We should be grateful for it."

So if science is not analogous to math, what is? It must be a discipline that fundamentally does not rely on observation or data. The social sciences must be disqualified, and even disciplines like history fall by the wayside. Though history is full of imaginings, they are grounded in texts written by people and artifacts belonging to the physical world. Math is also fundamentally different from disciplines in which analysis and interpretation of texts is the primary occupation. These disciplines are not rooted in the physical world, but they do have roots in texts. Though good criticism is creation in its own right, ultimately its analysis must come back to the text in question. Math, on the other hand, has no such grounding in texts.

Yet a work of art, like a mathematical theorem, stands alone as an object of free perception. Like mathematics – and unlike the social and natural sciences – art has no obligation to consult or even make reference to the real world, except in a few pursuits like portrait and landscape painting. A sculpture, though it may incorporate forms inspired by the physical world, does not depend in an essential way on the physical world for its legitimacy and power.

Much has been written on what motivates artists to create. Though the reasons are as varied as artists themselves, there exists a common desire to express something personal and ineffable, something that inspires wonder. Mathematicians, too, tend to be motivated by wonder. Recently I discussed with my advisor one of his early mathematical interests. How many people in the world, I asked, really paid attention to his work, not just his results but to his methods and arguments? He guessed about half a dozen – adding, "After a certain point, you're only doing math because it pleases you." What's pleasing is the sense of amazement provoked by math's surprising conclusions and clever turns of logic. The great questions of mathematics – the kind that draw people to math in the first place – are called great not because they may lead to applications, but because they captivate the imagination. They inspire wonder and delight. One could say, they are beautiful. Understanding the prime numbers, for instance, is a mathematical holy grail. However, notwithstanding a recent application to cryptography, the only real reason to investigate them is a sense of awe about their mysteries. Mathematics, like art, is generally done for its own sake.

Another similarity is the way work in each discipline is judged. If a work of art has the capacity to impart wonder and delight – if it is beautiful – it stands a good chance of becoming well-known. Although a mathematical theorem sometimes garners recognition through its applications to other areas of math, the purpose of these applications is nearly always to address one of the great questions. Individual arguments may be judged as "elegant" when they marshal the minimum possible tools for the task and are extremely and cleverly succinct. I believe this is the "stern perfection" of which Russell spoke. This idea of beauty, having nothing to do with the senses, is much like the beauty of poetry written in strict form. It resides in the artist's skill in working through and against the formal structure that is at once obstacle and inspiration. That Irish poet Dylan Thomas was able to say so much in his villanelle "Do Not Go Gentle into That Good Night" is a marvel; the strictness of the form amplifies his words as free verse could not have.

Yet despite their deep similarities, mathematics and art are hardly twins. First, art is perceived through the five physical senses, and good art brilliantly exploits this sensual perception. Though art may not be truly fathomed until understood intellectually, it makes subtle and profound use of the senses. In contrast, to say anything meaningful about math, one must grasp the concepts abstractly. Neither sight nor any other sense is the primary mode of mathematical understanding. Secondly, art emphasizes interpretation. While interpretation certainly exists in math – among ten mathematicians there are inevitably ten different ideas of what notions are beautiful – it's not part of mathematicians' principal business.

Let me close with a question. I have argued that math and art are profoundly similar, far more so than math and science. Yet I have also touched on the "unreasonable effectiveness of math" in application to the sciences. In what discipline or pursuit, then, might art be unreasonably effective?