

SEARCHING FOR GRAVITATIONAL WAVES

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Abstract

The search for gravitational radiation has reached a new era as the Laser Interferometer Gravitational Wave Observatory (LIGO) has reached its initial target sensitivity. Other similar interferometric detectors are also approaching their design goals. There is presently vigorous activity in the gravitational radiation community in the search for signals. In this paper we review the status of the interferometric detectors, and describe the efforts to find various types of astrophysically produced signals.

1 Introduction

It has been a long battle for researchers in the gravitational radiation detection field, but we sense that a new era is upon us. The Laser Interferometer Gravitational Wave Observatory (LIGO) ^{1, 2, 3)} is at its initial target sensitivity, and the detection of an event could come at any time. It will be a great day for physics when gravity waves are finally directly detected, but it will also be the birth of a new way of observing the universe and conducting astrophysics. The observations of expected sources should be dramatic; supernovae, pulsars, the inspiral of binary neutron stars followed by black hole formation, or even the stochastic background from the Big Bang. Members of the LIGO Scientific Collaboration (LSC) are enthusiastically working to make gravitational radiation detection a reality. Around the globe a world-wide network of detectors is coming on-line; VIRGO in Italy ^{4, 5)}, GEO in Germany ⁶⁾ (GEO researchers are also formally part of the LSC), and TAMA in Japan ⁷⁾ are operating alongside LIGO in the quest for gravity wave detection. These ground based laser interferometers are sensitive to gravitational radiation in the frequency band from 40 Hz up to 8 kHz.

The existence of gravitational radiation was predicted by Albert Einstein ⁸⁾, and confirmed through observations on the binary pulsar PSR 1913+16. This binary system was discovered in 1974 by Joe Taylor and Russell Hulse ⁹⁾, and subsequent observations by Taylor and Joel Weisberg ¹⁰⁾ have shown that the decay of the orbit matches perfectly with what is predicted via energy loss by gravitational radiation emission. Gravitational radiation is a consequence of general relativity. Accelerating masses can produce a ripple in space-time; a gravitational field that propagates away at the speed of light. Gravitational radiation, like electromagnetic radiation, carries energy away from the source; it also carries information about the source. Gravitational wave detectors, like LIGO and others, hope to observe gravitational radiation produced by astrophysical sources. The observation of these gravitational waves will provide information about the astrophysical event. LIGO, Virgo, GEO and other detectors will not be just gravitational radiation detectors, they will also be the new generation of astronomical observatories. It is possible that some sources of gravitational radiation may not emit electromagnetic radiation; for example, imagine the oscillations of a newly formed black hole. Other sources, like a supernova, will likely emit both electromagnetic and gravitational radiation, and the observation of the gravity waves in coincidence with electromagnetic observations could give new insight about the source. Electromagnetic observations of the universe are done with radiation having frequencies above 10 MHz. On the other hand, gravitational radiation observations will be from frequencies below 10 kHz; this should provide very different information about

the universe. Since gravitational radiation is extremely weakly interacting, any waves produced will traverse the universe without being scattered or absorbed; this gives another unique opportunity for scientists to *see* new phenomena in our universe.

Gravitational radiation has the effect of stretching and contracting space-time. Like its electromagnetic counterpart, the effect of gravitational radiation is transverse to the direction of propagation. Gravitational radiation also has two polarizations. For a linear polarized wave with amplitude h , two points separated by a distance L will find their distance separation increased by $\Delta L = hL$; this dimension has been stretched. Conversely, the dimension perpendicular to the direction of the waves propagation and the dimension that has stretched will find two points, initially separated by L , contracted by an amount $\Delta L = -hL$. The gravity wave produces a *strain* on space. The amplitude of a wave of this polarization is referred to as h_+ . As the wave propagates through the plane of observation the stretching will change to contraction along the first axis, and vice versa on the other. The other polarization, with amplitude h_\times will have a similar effect, but on two axes that are 45° from the “+” polarization. In fig.1 the stretching and contracting of space-time, produced by a passing gravity wave, is displayed.

The problem with gravitation radiation, however, is that it is extremely weak. The perturbation on flat Minkowski space-time, produced by a gravity wave is

$$h_{\mu\nu} = \frac{2G}{c^4 r} \frac{d^2 I_{\mu\nu}}{dt^2}, \quad (1)$$

where r is the distance to the source, and $I_{\mu\nu}$ is the source mass quadrupole moment. The small value for the gravity wave amplitude is a consequence of the G in the numerator and c^4 and r in the denominator. Only events with extreme mass (and hence astrophysical) will produce a signal large enough for possible detection. As an example, consider a pair of neutron stars in the final stage of their decaying orbit. The amplitude of the gravity wave would be

$$h \approx \frac{4\pi^2 G M R^2 f_{orb}^2}{c^4 r}. \quad (2)$$

Taking the individual neutron star masses as $M = 1.4M_\odot$, the orbital frequency of $f_{orb} = 400Hz$ when the masses are separated by $R = 20km$, and a distance to the source of $r = 10^{23}m = 3.24Mpc$, the resulting gravity wave would have a strain amplitude of only $h = 10^{-21}$.

Researchers in the LSC are actively searching LIGO data for different types of gravitational radiation signals; upper limit results have been presented for searches for pulsar signals (11, 12, 13), burst events (14, 15, 16, 17, 18), binary inspiral signals (19, 20, 21, 22, 23) and the stochastic gravitational wave background (24, 25). With LIGO reaching its initial target sensitivity

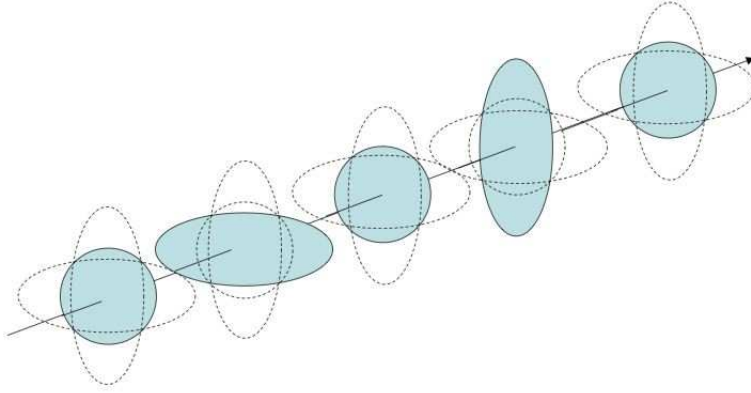


Figure 1: *An example of the stretching and contracting of space-time caused by a gravitational wave.*

goal it is possible that a detection could come very soon. Coalescing binaries containing neutron stars or black holes promise to be one of the cleanest and most promising sources of detectable radiation²⁶⁾. Observation of inspiral events could provide important information on the structure of neutron stars^{27, 28)}. Even cosmological information can be extracted from the observation of inspiral events^{29, 30, 31, 32)}. The characteristics of radiation in the post-Newtonian regime will provide insight into highly non-linear general relativistic effects, such as the observation of the formation of a Kerr black hole as the binary system decays^{31, 33, 34)}. An axi-symmetric spinning neutron star will not produce gravitational waves, but a number of mechanisms have been proposed that are capable of producing quasi-periodic gravitational waves from biaxial or triaxial neutron stars^{35, 36)}. Similarly, an asymmetry in a supernova should produce a burst signal. The Big Bang itself will be a source for the stochastic gravitational radiation background.

2 Laser Interferometric Gravity Wave Detectors

The quadrupole stretching and contracting of space-time by a gravity wave will hopefully be detected by a Michelson interferometer, where the arms are at right angles to one another. This is the strategy for detection by LIGO, Virgo, GEO and TAMA. Imagine one such interferometer, with arm of length L . Coherent laser light will initially strike a beam-splitter, then pass down each arm. A mirror at the end of each arm will reflect the light back toward the beam-splitter. If both arms have equal lengths, no light will exit from the *output port* of the beam-splitter. However, if an optimally oriented and polarized gravity wave, with amplitude h , strikes the interferometer from above, then one arm will be increased in length by $\Delta L = hL$, while the other will decrease by the same amount. With the unequal arm lengths there will now be some light exiting the interferometer from the output port.

However, the small size of the gravity wave makes this experiment difficult. For a wave with amplitude of $h = 10^{-22}$, and an arm length of $L = 4\text{km}$, the resulting change in the arm length will be only $\Delta L = 4 \times 10^{-19}\text{m}$, less than a thousand times the size of a nucleus. In essence, every photon circulating in the interferometer is a *meter stick*, and the repeated measurements with numerous photons allows this small distance to be experimentally determined. The photons provide an average relative distance difference measurement of the arm lengths. In fact, increasing the number of photons by raising the laser power has the effect of improving the detection sensitivity, with the signal to noise ratio (SNR) depending on the laser power P as $SNR \propto P^{-1/2}$ in the frequency regime where laser *shot noise* dominates.

Detectors, like LIGO and others, employ numerous engineering techniques in order to allow these small distance measurements. One technique is to replace the simple Michelson interferometer with one where the arms are Fabry-Perot cavities. The light is then *stored* in the arms, and the effective arm length of the detector increases, thereby making the detector more sensitive to smaller waves. For LIGO, these Fabry-Perot cavities increase the effective distance by ≈ 50 . The four mirrors associated with the two Fabry-Perot cavities are suspended by wires. This allows a gravity wave to freely move the mirrors; the suspension apparatus also acts as a filter for seismic noise that will try to move the mirrors. A simple drawing of a gravity wave detector made of a Michelson interferometer with Fabry-Perot cavity arms is displayed in fig. 2. In reality, the gravity wave interferometers are far more complex than what is displayed in this figure. For example, LIGO and other detectors employ a technique called *power recycling*; the laser light that would normally exit the system and head back toward the laser is reflected back (recycled) in. Power recycling increases the amount of light circulating in the interferometer, thereby increasing the sensitivity.

The LIGO observatories consist of a 4 km arm length interferometer in

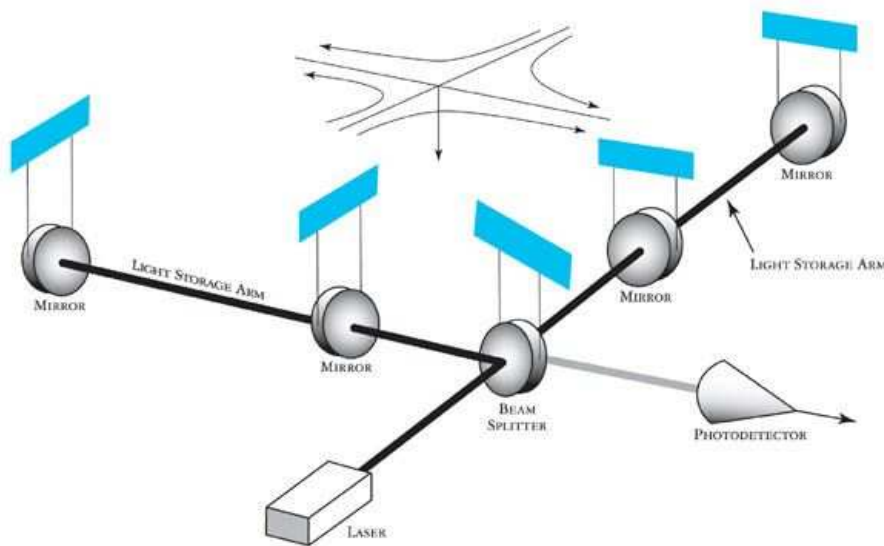


Figure 2: A Michelson interferometer with Fabry-Perot cavities in the arms. A passing gravity wave will stretch one arm and contract the other. The mirrors are suspended by wires like a pendulum; this allows free motion when a gravity wave passes, and also provides isolation from seismic noise. Thermal noise in the mirrors and wires will cause deleterious motion of the mirrors. Photon counting statistics will also create noise at the photodetector output, namely shot noise.

Livingston, Louisiana, a 4 km interferometer in Hanford, Washington, and a 2 km interferometer also at Hanford. The 2 km and 4 km interferometers at Hanford are in the same vacuum system. The GEO interferometer has a 600 m arm length, and is located outside of Hannover, Germany. Virgo has an arm length of 3 km, and is located in Cascina, Italy, near Pisa. These interferometers use stabilized lasers operating at 1064 nm. LIGO has now reached its target design sensitivity, and GEO and Virgo are expected to do the same in the near future.

LIGO and the LSC have gone through a number of *scientific runs* where data was collected and analyzed. So far, LIGO has completed four science runs (S1-S4) and is now in its fifth science run, S5. Between these runs the

interferometer performance was improved through commissioning work. In fig. 3 the evolution of LIGO's noise strain sensitivity is displayed for the science runs S1-S5. Also displayed in this figure is LIGO's design sensitivity goal; this is a summary of all the limiting noise sources. LIGO has more than met its design goal of $h_{rms} < 10^{-21}$, and has achieved a strain sensitivity of $h(f) < 3 \times 10^{-23} \text{ Hz}^{-1/2}$ at 200 Hz, and $h_{rms} \approx 10^{-21}$ within a bandwidth of 100 Hz. At the time of this writing, LIGO was in the middle of its S5 run, with the goal to acquire a year of coincident data. S5 commenced in November of 2005. The triple coincident duty cycle has been about 45%. A measure of the interferometer's performance is the sensitivity range to an inspiral of a binary neutron star system. In S5 the LIGO 4 km interferometers have a sensitivity range for $1.4M_{\odot}$ - $1.4M_{\odot}$ binary inspirals out to a distance of 14.5 Mpc. This number accounts for averaging over all sky locations and orientation angles for the binary system. The LIGO 4 km interferometers are sensitive to an optically oriented (directly overhead, with binary orbital plane parallel to the interferometer plane) neutron star binary inspiral out to more than 33 Mpc. The LIGO 2 km interferometer has a sensitivity that is poorer by a factor of 2.

3 LIGO's Observational Results

Even though LIGO is at its initial design sensitivity, a gravitational radiation signal has yet to be detected. LIGO is searching for various types of events. In the absence of a detection, LIGO will place upper limits on the frequency of occurrence for impulsive signals, or an upper limit on the strength of steady signals.

The coalescence of compact binary systems promises to provide a *clean* signal, whose form is well-understood. The decay of the orbit is a consequence of gravity wave emission. As the orbit decays, the orbital frequency will increase, producing a chirp-like gravity wave signal that increases in frequency and amplitude with time. The systems are accurately modeled, with signals only depending on the masses and spins of the compact objects. LIGO will be sensitive in its operating frequency band to binary inspiral signals from neutron star and black hole systems with individual component masses below $20M_{\odot}$. Matched filtering is used by LIGO to search for binary inspiral events. Using the data from LIGO's S2 run, it was possible to set an upper limit on the neutron star coalescence rate of less than 50 per year per Milky Way equivalent galaxy ²⁰⁾. LIGO has also conducted searches for binary inspiral signals from primordial black holes ($0.2 - 1.0 M_{\odot}$) in the halo of our galaxy ²¹⁾, plus more massive black hole systems where component masses are in the $3.0 - 20 M_{\odot}$ range ²²⁾.

The violent explosion of a star, or a supernova, will produce gravitational radiation if the process is non-symmetric. This will produce a *burst* event, or

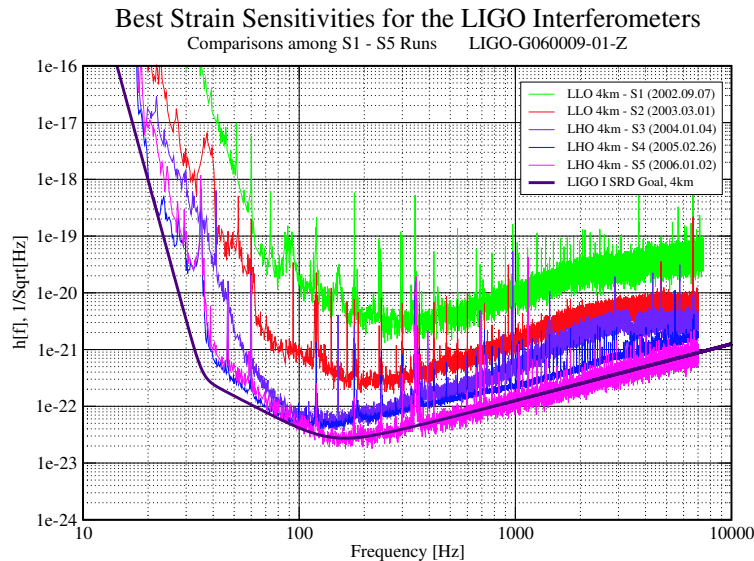


Figure 3: *The progression of the LIGO noise strain sensitivity for the science runs, S1-S5, conducted to date. The solid line shows LIGO's design target sensitivity, and takes into account all noise sources (seismic, thermal, laser shot noise, etc.).*

one that is short in time, but relatively large in amplitude. Other possible sources may also produce intense gravity signals that are short in time. In addition to supernovae (and their associated gamma ray bursts) there could be burst-like signals from the final merger of neutron star or black hole binary systems, instabilities in nascent rotating neutron stars, or kinks and cusps on cosmic strings. Unfortunately, the exact gravitational waveform for most of these types of events are not well-known (the string waveform is, in fact, known), so general burst search templates need to be employed. LIGO conducted a search for a signal associated with gamma ray burst GRB0303029¹⁵⁾. The burst occurred while LIGO was acquiring data during its second science run S2. No event associated with the gamma ray burst was observed, and an upper limit was placed on the size of a possible gravity wave associated with this gamma ray burst event. Near the LIGO interferometers' most sensitive frequency region, around 250 Hz, the root-sum-square (RSS) gravitational-wave strain sensitivity

for optimally polarized bursts was better than $h_{RSS} = 6 \times 10^{-21} \text{ Hz}^{-1/2}$.

Rapidly spinning neutron stars, or pulsars, could be sources of gravitational radiation. In order for radiation to be produced the neutron star will need to be non-axisymmetric in shape. This type of gravitational radiation will be a nearly perfect sinusoidal signal. One must still account for Doppler shifts due to the motion of the Earth, and changes in the interferometers' response as the Earth rotates and orbits about the sun. Radio observations can help the search as this provides sky location, rotation frequency and spin down rate. The gravity wave frequency will be at twice the rotation frequency. All sky searches for unknown sources are also possible. In the absence of a signal it is still possible to produce meaningful astrophysical results. One can attribute the spin down rate of pulsars as being due to the energy loss from gravity wave emission. This then makes a statement as to the degree of asymmetry on the pulsar. Upper limits on the strength of gravity waves from pulsars are providing limits on the asymmetry that are approaching the spin down limits. LIGO has published a series of results on the upper limits of signal strength for various pulsar signals (11, 12, 13). Using the S2 data 28 pulsars were studied, and limits on the strain signal strength as low as $h \approx 10^{-24}$ were achieved; this correspond to limits on pulsar ellipticity as low as 10^{-5} (12). The pulsar gravity wave signal limits set by LIGO with its S2 data are displayed in fig. 4.

The final class of signals that LIGO is actively trying to detect concerns a stochastic gravitational wave background. The Big Bang will produce a stochastic background of gravity waves, analogous to the electromagnetic cosmic microwave background. This would seem to be a background noise in each detector, but the signal could be extracted through a correlation of the outputs of two detectors (37). A background could also be produced after the Big Bang through the addition of signals from binary systems or supernovae throughout the universe. LIGO is actively searching for the stochastic background (24, 25), and setting limits on its strength. The magnitude of the stochastic background is usually described by the gravity wave energy density per unit logarithmic frequency, divided by the critical energy density to close the universe, $\Omega_{gw}(f)$. Using the S3 data LIGO was able to set a limit on the stochastic gravity wave energy density of $\Omega_{gw}(f) < 8.4 \times 10^{-4}$ in the frequency band from 69 Hz to 156 Hz (25). An example of LIGO's ability to constrain $\Omega_{gw}(f)$ with the S3 data is displayed in fig. 5.

The limits on $\Omega_{gw}(f)$ that LIGO may set with its S4 and S5 data could approach the limit set on it via nucleosynthesis (38). If the energy density of gravitational radiation at the time of nucleosynthesis were too large it would affect the ratio of light nuclei production. The Big Bang nucleosynthesis model, and observations of light element abundances, constrain the total gravity wave

energy density at the time of nucleosynthesis to

$$\int \Omega_{gw}(f)d(\ln f) < 1.1 \times 10^{-5}(N_\nu - 3), \quad (3)$$

where N_ν is the number of relativistic species at the time of nucleosynthesis. With an upper bound of $N_\nu - 3 < 1.4$ ³⁹⁾, the nucleosynthesis limit then becomes $\int \Omega_{gw}(f)d(\ln f) < 1.5 \times 10^{-5}$. LIGO researchers are hopeful that their data will soon set a limit of $\Omega_{gw}(f)$ below this important level.

4 Discussion

This is an exciting time for gravitational radiation research. With LIGO reaching its target sensitivity there is great hope that a detection could occur at any time. GEO and Virgo are also approaching their target sensitivity, and the plan for the future is to have the LSC and Virgo collaborate and analyze all data together ^{40, 41, 42)}. Increasing the number of observatories working in coincidence raises the probability of a detection ⁴²⁾.

LIGO's S5 data run will continue until sometime in 2007. After some further improvements are made to the interferometers' performance LIGO will likely commence with another science run, starting around 2009. After S6, sometime around 2010, LIGO will initiate an upgrade to *Advanced LIGO*. From the time after S5 through to the Advanced LIGO commissioning work LIGO, GEO, and Virgo will likely try to coordinate activity so that two interferometers are always operating in coincidence.

Advanced LIGO should present an amazing increase in technology and detection sensitivity. The noise sensitivity will be improved over the current LIGO sensitivity by a factor of 10. Advanced LIGO will be able to *see* neutron star binary inspirals out to a distance of 350 Mpc (assuming optimal alignment), with an expected event rate between 2 per year to as many as 3 per day (there is much uncertainty in these rates). Black hole - black hole binary inspirals should be detectable out to a distance of 1.7 Gpc, with an event rate between 1 per month to 1 per hour. Black hole - neutron star binary inspirals will be detectable out to 750 Mpc, with an event rate between 1 per year to 1 per day.

It is also worth noting that there is currently much work and research being conducted in the planning and design for the space-based gravitational radiation detector, LISA, or the Laser Interferometer Space Antenna ⁴³⁾. This joint project between NASA and the ESA will be sensitive to gravitational radiation in the frequency range from $10^{-2}mHz$ to $100mHz$, and will hopefully commence acquiring data around 2015. The era of gravitational radiation astronomy is starting, and should be a very active field of research for years to come.

5 Acknowledgments

LIGO Laboratory and the LIGO Scientific Collaboration gratefully acknowledge the support of the United States National Science Foundation (NSF) for the construction and operation of the LIGO Laboratory and for the support of this research. N.C. also acknowledges support via NSF grants PHY-0244357 and PHY-0553422, and the Fulbright Scholar Program. Gabriela González provided much material for this presentation, and her help is gratefully acknowledged.

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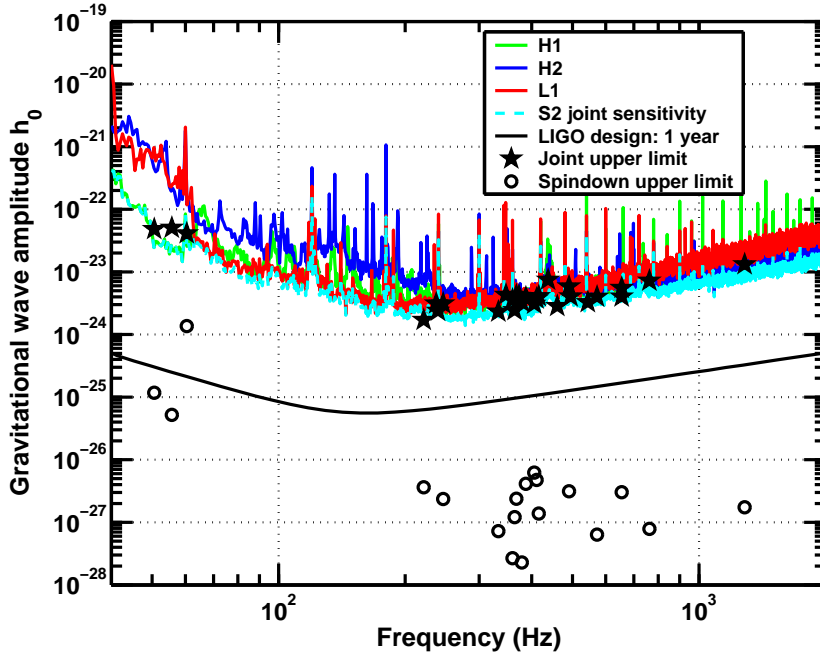


Figure 4: As presented in ¹²⁾, the upper curves are the h_0 amplitudes detectable from a known generic source with a 1% false alarm rate and 10% false dismissal rate for single detector analyses and for a joint detector analysis. All the curves use typical S2 sensitivities and observation times. H1 and H2 are the 4 km-arm and the 2 km-arm detectors located in Hanford WA. L1 is the 4 km-arm detector situated in Livingston Parish LA. Lower curve: LIGO design sensitivity for 1 yr of data. Stars: upper limits found in this paper for 28 known pulsars. Circles: spindown upper limits for the pulsars with negative frequency derivative values if all the measured rotational energy loss were due to gravitational waves and assuming a moment of inertia of 10^{45} g cm².

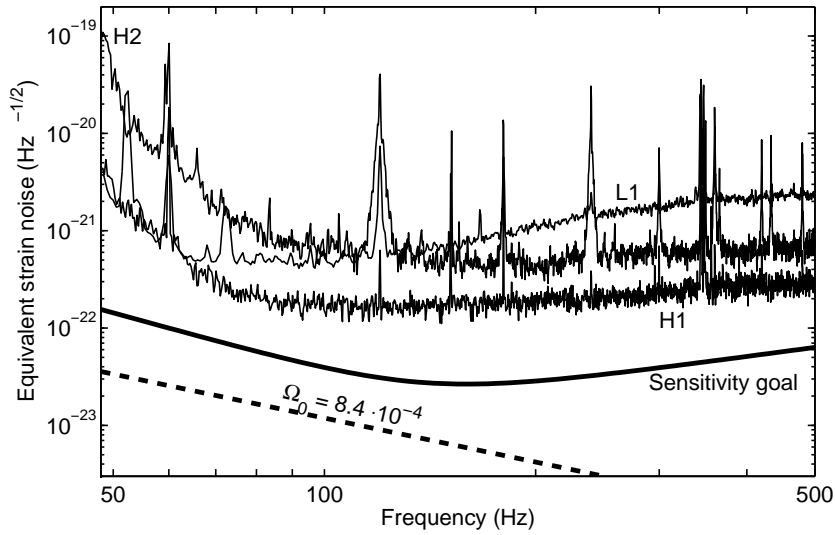


Figure 5: From ²⁵⁾: Reference sensitivity curves during the LIGO S3 data run, in terms of equivalent strain noise density. Also shown is the strain noise level corresponding to the upper limit found in the S3 analysis, $\Omega_0 = 8.4 \times 10^{-4}$, and the strain noise goal for the two 4-km interferometers.