Fill in the missing steps of this derivation from your textbook (in the space provided between equations). Do not just use math symbols but also include explanatory text in English. If you need to see an example, look for other derivations in the book.

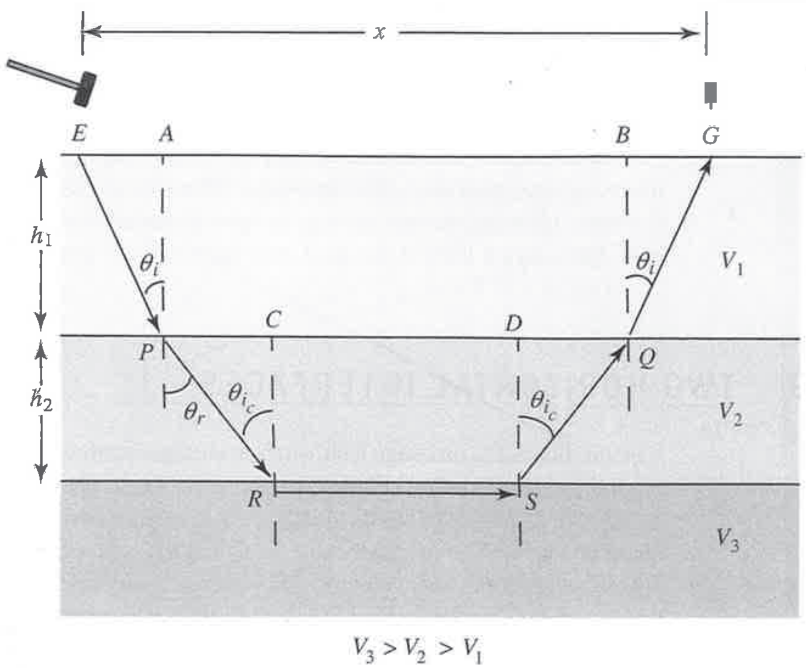

FIGURE 3.9 Diagram illustrating symbols used in derivation of time of travel for ray critically refracted along the second interface in a three-layer case.

Everything we now derive follows the same procedure as for the single interface. The total travel-time equation is

$$\text{Time} = \frac{EP}{V_1} + \frac{PR}{V_2} + \frac{RS}{V_3} + \frac{SQ}{V_2} + \frac{QG}{V_1}. \tag{3.27}$$

As before

$$EP = QG = \frac{h_1}{\cos\theta_i} \quad \text{and} \quad PR = SQ = \frac{h_2}{\cos\theta_{i_c}}.$$

Also,

$$EA = BG = h_1 \tan\theta_1 \quad \text{and} \quad PC = DQ = h_2 \tan\theta_{i_c} \quad \text{so that}$$

$$RS = x - 2h_1 \tan\theta_i - 2h_2 \tan\theta_{i_c}. \tag{3.28}$$

These relationships can be substituted into Equation 3.27 so that our travel-time equation becomes

$$\text{Time} = \frac{2h_1}{V_1\cos\theta_i} + \frac{2h_2}{V_2\cos\theta_{i_c}} + \frac{x - 2h_1\tan\theta_i - 2h_2\tan\theta_{i_c}}{V_3}. \tag{3.29}$$

It's worthwhile to present the major steps in simplifying this equation; but because the steps are similar to those used to arrive at Equation 3.14, they are presented without comment. The same identities that we used previously are adequate.

$$\text{Time}_{RS} = \frac{x}{V_3} - \frac{2h_1\tan\theta_i}{V_3} - \frac{2h_2\tan\theta_{i_c}}{V_3} \tag{3.30}$$

$$\text{Time} = \frac{x}{V_3} + \frac{2h_1}{V_1\cos\theta_i} + \frac{2h_2}{V_2\cos\theta_{i_c}} - \frac{2h_1\sin^2\theta_i}{V_1\cos\theta_i} - \frac{2h_2\sin^2\theta_{i_c}}{V_2\cos\theta_{i_c}} \tag{3.31}$$

$$\text{Time} = \frac{x}{V_3} + \frac{2h_1 - 2h_1\sin^2\theta}{V_1\cos\theta_i} + \frac{2h_2 - 2h_2\sin^2\theta_{i_c}}{V_2\cos\theta_{i_c}} \tag{3.32}$$

$$\text{Time} = \frac{x}{V_3} + \frac{2h_1\cos\theta_i}{V_1} + \frac{2h_2\cos\theta_{i_c}}{V_2} \tag{3.32}$$

and, finally

$$\text{Time} = \frac{x}{V_3} + \frac{2h_1(V_3^2 - V_1^2)^{\frac{1}{2}}}{V_3V_1} + \frac{2h_2(V_3^2 - V_2^2)^{\frac{1}{2}}}{V_3V_2}. \tag{3.33}$$

Once again we finish with an equation for a straight line. As you probably have observed by now, if we again take a derivative—voilà!

$$\frac{dt}{dx} = \frac{1}{V_3}. \tag{3.34}$$