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Quantum Delta-Kicked Rotor: Experimental Observation of Decoherence

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We report on the experimental observation of environment induced decoherence in the quantum delta-kicked rotor. Ultracold cesium atoms are subjected to a pulsed standing wave of near resonant light. Spontaneous scattering of photons destroys dynamical localization thereby giving rise to a quantum diffusion, which approaches the classical diffusion with an increasing degree of decoherence. This tendency is enhanced by a stronger stochasticity in the underlying classical system. A comparison with theoretical predictions is presented. [S0031-9007(98)06007-4]

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It is nowadays widely accepted that sensitive dependence on initial conditions does not occur in closed—and generic—single particle quantum systems. Nonetheless, because of the quantum classical correspondence (QCC) principle, quantum mechanics must contain the macroscopic limit and thus be able to describe classical chaos. Employing solely the usual semiclassical limit $\hbar \rightarrow 0$ is not entirely satisfactory. In this world, $\hbar$ is not equal to zero. No matter how small it is, after the Ehrenfest time quantum effects start to play. This time might be very long in an "integrable world," but chaotic systems are well known to develop highly complex phase space structures of order $\hbar$ in logarithmically short times $\sim \ln(1/\hbar)$. According to Zurek *et al.* [1], this difficulty is eliminated by realizing that it is not possible to isolate macroscopic systems from their environment. The coupling of a quantum system to extraneous degrees of freedom destroys the quantum coherences. This explains why macroscopic superposition states are not observed and reconciles the semiclassical limit of quantum mechanics with classical dynamics.

The purpose of this Letter is to experimentally investigate environment induced decoherence of dynamically localized states in the quantum δ-kicked rotor (Q-DKR) [2]—a topic which has already been addressed in various theoretical studies [3–5]. Our experimental system consists of a gas of ultracold cesium atoms which are subjected to a pulsed standing wave of near resonant light. In this atom optics realization of the Q-DKR [6], the quantum dynamics becomes susceptible to the decohering effects of spontaneous emission by decreasing the laser-atom detuning. The "environment" in this case is, of course, the vacuum fluctuations. Measurements of the atomic momentum distribution, as a function of time and detuning, provide a direct examination of the loss of coherence in quantum system. Although we do *not* perform the semiclassical limit $\hbar \rightarrow 0$ in the laboratory, which would be mandatory in order to demonstrate QCC experimentally, we will see that *some* dynamical features characteristic for the classical DKR (C-DKR) are partially restored in the Q-DKR by increasing the coupling with the environment.

To model our system, we first note that although spontaneous emission plays a key role in our experiment, the detuning $\delta_L = \omega_L - \omega_0$ (where ω_L and ω_0 denote the optical and the atomic transition frequency, respectively) will typically be 2 orders of magnitude larger than the natural linewidth. We therefore neglect spontaneous emission for the moment. Then, adopting the notation which has been used in [6], we write the Hamiltonian in dimensionless form as

$$H = \frac{\rho^2}{2} - k \cos\phi \sum_{n=1}^{N} f(t-n), \qquad (1)$$

where $f(t)$ specifies the temporal shape of the "kicks." We will not explicitly give the relations between the dimensionless and the "real" parameters as they are the same as in [6] with one exception: Instead of $\Omega_{\rm eff} = \Omega^2/\delta$, we write $\Omega_{\text{eff}} = \Omega^2(s_{45}/\delta_{45} + s_{44}/\delta_{44} + s_{43}/\delta_{43})$, where the terms in brackets take account of the different dipole transitions between the relevant hyperfine levels in cesium ($F = 4 \rightarrow F' = 5, 4, 3$). In our simulations we assumed equal populations of the Zeeman sublevels, yielding numerical values for the s_{4j} of $s_{45} = 11/27$, $s_{44} = 7/36$, and $s_{43} = 7/108$; δ_{4j} are the corresponding detunings. Note, however, that different magnetic sublevels will experience different ac stark shifts. For the smallest detuning used in this work a 5% spread in the coupling strength results. Whereas Moore *et al.* [6] used Gaussian pulses to model their experimental situation, the shape of our pulses is much closer to rectangular (dimensionless pulse width α). Then, for an infinite train of kicks, the Hamiltonian (1) can alternatively be written as

$$H = \frac{\rho^2}{2} - \kappa \sum_{m=-\infty}^{\infty} \text{sinc}(\pi\alpha m)\, \cos(\phi - 2\pi m t), \quad (2)$$

where we have defined $\kappa = \alpha k$ such that in the limit $\alpha \rightarrow 0$, $k \rightarrow \infty$ (αk finite) Eq. (2) reduces to the usual DKR Hamiltonian $H = \rho^2/2 - \kappa \cos\phi \sum_{n=-\infty}^{\infty} \delta(t - n)$. The sinc function is defined as $\text{sinc}(x) = \sin(x)/x$. Interpreting Eq. (2) classically, the fundamental resonances are located at $\rho_m = 2\pi m$ with widths given by $\delta\rho_m = 4\sqrt{\kappa\, \text{sinc}(\pi\alpha m)}$ [7]. Employing the Chirikov overlap criterion, we derive the condition $\kappa\, \text{sinc}(\pi\alpha m) \geq \pi^2/4$ for the destruction of the last Kolmogorov-Arnold-Moser (KAM) torus in the interval $[\rho_m, \rho_{m+1}]$. From this, we can immediately infer that there will be a *regular* region in phase space around $m_R \approx 1/\alpha$ even for very large κ. For smaller κ values, there will be a transition (as one goes to larger m values) to a mixed phase space structure around some critical $m_C < m_R$. In this work, the experimental parameters that imply the smallest degree of chaoticity are $\kappa = 17$ and $\alpha = 0.03$, yielding a critical *integer* momentum of $n_C \equiv \rho_C/\bar{k} = 2\pi m_C/\bar{k} \approx 87$. We have actually observed these KAM boundaries, so it was easy to make sure that they did not affect the measurements.

Our experimental setup is very similar to that of Moore *et al.* [6]. Approximately 10^5 cesium atoms are initially trapped and laser cooled in a standard magneto-optic trap (MOT). The atomic gas' temperature after a 20 ms cooling phase (by increasing the detuning and decreasing the intensity of the trapping beam) is slightly below 10 μK. The position distribution of the trapped atoms has a FWHM of 200 μm. The modulated periodic potential is generated by a third laser diode. The beam passes through an 80 MHz acousto-optic modulator (AOM) and a single mode optical fiber. The collimated beam with a measured waist of $2\sigma = 1$ mm is then retroreflected from a mirror outside the vacuum cell to generate the one-dimensional potential, which is temporally modulated via the rf supply to the AOM. Taking reflection losses at the windows of the containing glass cell into account, the Rabi frequency in the center of the MOT is $\Omega/2\pi = 310$ MHz. The finite widths of the beam waist and the atomic cloud entail a reasonably narrow distribution of κ with rms spread of 10% and $\kappa_{\text{mean}} \approx 0.9\kappa_{\text{max}}$, where κ_{max} is the kicking strength on the beam axis. In the following, when specifying κ, this always refers to κ_{mean}. The pulse spacing used is $T = 20$ μs ($\bar{k} = 2.1$). Note that both the Rabi frequency Ω and the pulse spacing T are held constant throughout the whole work. Varied are only the pulse width ($\alpha T \approx 90-580$ ns) and the detuning ($\delta_{45}/2\pi \approx 0.62-4.0$ GHz to the blue of the $F = 4 \rightarrow F' = 5$ transition). The latter is monitored by overlapping the kicking beam with the trapping beam and measuring the beating frequency using a fast photodiode and a spectrum analyzer. After trapping and cooling, the trap is turned off leaving the atoms in the $F = 4$ ground state. They have at most a 1:6 chance per spontaneous scattering to fall into the $F = 3$ ground state. In order not to let them steal away, we leave the repumping beam on during the experiment. This produces a small additional heating due to incoherent transitions back to the $F = 4$ state, which, however, is of no importance as for our parameters heating effects are negligible altogether. According to the work of Dyrting [3], the relative contribution to the energy diffusion arising from recoil heating (as opposed to diffusion due to decoherence) is smaller than $1/(2E_{\text{sat}})$ where E_{sat} is the saturation energy (see below). In this work, heating contributes less than 1% to the overall diffusion. Its insignificance is also supported experimentally: By blocking the retroreflected beam, thus turning the standing wave into a traveling wave, the momentum diffusion effectively disappears. To measure the atomic momentum distribution we use a time-of-flight technique with a "freezing molasses" [6]. The main information extracted from the momentum distribution is

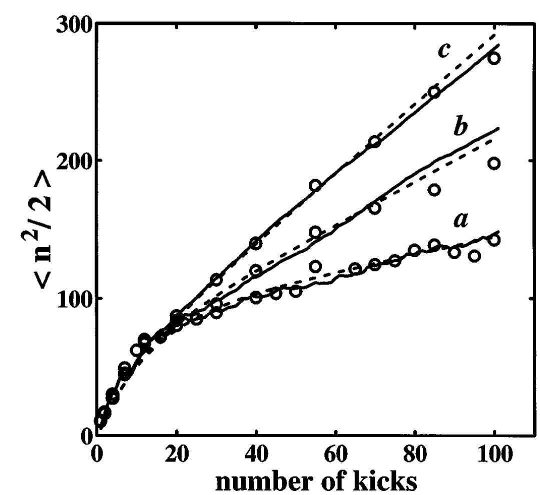

FIG. 1. Kinetic energy $\langle n^2/2 \rangle$ as a function of number of kicks. The experimental (circles) and simulation (solid lines) results are for $\kappa = 12.5$, $\bar{k} = 2.1$, and (a) $\eta = 0.76 \times 10^{-2}$, ($b$) $\eta = 2.3 \times 10^{-2}$, and ($c$) $\eta = 4.6 \times 10^{-2}$. The corresponding detunings are $\delta_{45}/2\pi = 4$, 1.3, and 0.62 GHz, respectively. The parameters for the analytical (dashed lines) traces are $D_0 = 13$ and $N^* = 14$ (η values as above).

its kinetic energy. Whereas its determination does not constitute a problem for small energies, the experimental uncertainties grow for higher energies due to the fact that a considerable portion of the energy is contained in the wings of the measured distributions. We estimate the systematic errors to be roughly 30% for the largest energies measured, whereas the relative errors are approximately 10%.

We now turn to the description of the experimental results. In the absence of spontaneous emission the atomic momentum distribution initially diffuses, followed by the onset of dynamical localization. Spontaneous emission introduces decoherence to the Q-DKR. This destroys dynamical localization and results in *quantum diffusion* (we use this term to refer to momentum diffusion *after* the quantum break time). Figure 1 displays the measured growth of the atoms' kinetic energy with time for different detunings. The initial diffusion rate is held constant by choosing smaller pulse widths for smaller detunings. For the three displayed traces, the probabilities for spontaneous emission per kick are $\eta = 0.76 \times 10^{-2}$, 2.3×10^{-2}, and 4.6×10^{-2}, respectively (with errors below 10%). The initial "classical-like" diffusion can clearly be distinguished from the quantum diffusion. Although 0.76×10^{-2} seems to be a small scattering probability, one can see that there is considerable quantum diffusion even in this case of large atom-laser detuning ($\delta_{45}/2\pi = 4.0$ GHz). It should be mentioned that Goetsch and Graham [3] claim that Moore *et al.* [8] might actually have seen decoherence due to spontaneous emission in their phase modulated standing wave experiment. Note that we cannot increase the detuning any further while maintaining a high chaoticity because of the limited power provided by the kicking beam laser diode. However, we would like to point out that we observe almost perfectly shaped exponential momentum distributions (the signal to noise ratio is typically 200:1) in spite of the nonzero quantum diffusion rate. This appears to be contradictory at first sight, as an exponential momentum distribution is the hallmark of localization. But a similar behavior has been found in the case of a phase modulated potential [5]. There it has been shown, based on analytic calculations, that in the case of a not too large quantum diffusion, the momentum distribution remains essentially exponential. Experimentally, delocalization reveals itself via energy diffusion rather than a transition from exponential to Gaussian line shapes. This is especially true when one considers a realistic signal to noise ratio of the charge-coupled device (CCD). The finite quantum diffusion, originating from less than one scattered photon over the whole time evolution, reflects the extreme vulnerability of quantum coherences, although the system we are dealing with is far from the semiclassical regime. As we shall see, the underlying classical chaoticity contributes much to this vulnerability. Also displayed in Fig. 1 are the analytic results (see below) and those of Monte Carlo wave function simulations. The latter were carried out by simply adding an interaction term $H_{\rm int} = -\zeta u \bar{k} \phi \sum_{n=1}^{N} \delta(t-n)$ to the Hamiltonian (1), where ζ is either 0 or 1, $\langle\zeta\rangle = \eta$, and $u\bar{k}$ is the recoil momentum projected onto the kicking beam axis (u chosen randomly from the interval $[-1, +1]$). We justify this procedure by the small degree of internal atomic excitation and by the fact that a mixing of internal and translational degrees of freedom does not alter the Q-DKR behavior significantly even if the excited state probability is large [9]. In Fig. 2, we show the dependence of the quantum diffusion coefficient defined as $\mathcal{D}_\infty \equiv \lim_{N\to\infty}\langle n^2\rangle/N$ on the rate of decoherence η. To gain some understanding for this dependence, we heuristically derive an analytical expression for $\mathcal{D}_\infty$ as follows. We first assume that one spontaneous scattering event causes complete decoherence between the atomic wave function and the Floquet states [5]. Then, realizing that the measured diffusion at a given instant will be a mixture of contributions from different atoms at different stages of their time evolution, the diffusion coefficient can be written as $\mathcal{D}_\infty = \sum_{k=0}^{\infty} \eta(1-\eta)^k D(k)$, where $D(k)$ is the time-dependent diffusion coefficient in the absence of spontaneous emission. Note that we have neglected heating effects. This formula is the same as Eq. (6.12) in Ref. [4], which was derived there using more rigorous arguments. Using $D(k) = D_0 \exp(-k/N^*)$ [4], we arrive at

$$\mathcal{D}_\infty = \frac{\eta N^* D_0}{1 + \eta N^*}, \tag{3}$$

where the parameters D_0 and N^* denote the initial diffusion coefficient and the crossover time, respectively ($N^* \gg 1$ has been used). Along similar lines, one can also derive an expression for the *time-dependent* diffusion coefficient and, by a summation over the number of kicks,

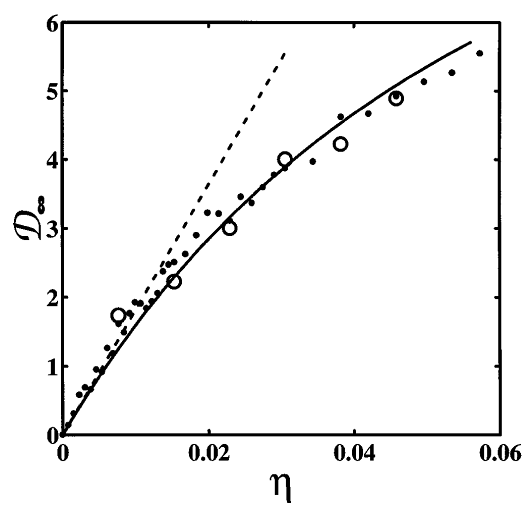

FIG. 2. Quantum diffusion coefficient $\mathcal{D}_\infty$ versus probability for spontaneous scattering per kick η. Experimental (circles) and simulations (dots) results for $\kappa = 12.5$ and $\bar{k} = 2.1$. The parameters for the analytical (solid line) traces are the same as in Fig. 1. The dashed line shows the perturbative behavior, i.e., Eq. (3) without saturation.

the time dependence of the kinetic energy $E \equiv \langle n^2/2 \rangle$. The final results can be expressed as $\mathcal{D}(N) = q^N D_0 + (1 - q^N)\mathcal{D}_\infty$ and $E(N) = \mathcal{D}_\infty N/2 + [(D_0 - \mathcal{D}_\infty)/2][(1 - q^N)/(1 - q)]$, where $q \equiv (1 - \eta)\exp(-1/N^*)$. Note that Eq. (3) is not compatible with the results for a quantum-kicked *particle* (which corresponds to the present situation) found in [4]. There, a power-law dependence $\mathcal{D}_\infty \sim \eta^{1/3}$ is predicted for small noise levels. This discrepancy arises from the different decoherence processes considered. In Ref. [4], decoherence progresses gradually and is caused not only by momentum diffusion but also by an associated spatial spreading, which accelerates the process. In the present situation, however, there is no time for such an interplay because the coherence is destroyed by a single event of spontaneous emission. Returning to Figs. 1 and 2, we find consistency between the measured data and the analytical expressions for $D_0 \approx 13$ and $N^* \approx 14$. These values imply a saturation energy in the absence of spontaneous emission of $E_{\text{sat}} \equiv \langle n^2/2 \rangle_{\text{sat}} = D_0 N^*/2 \approx 90$, which is roughly what one would infer from the experimental data displayed in Fig. 1. Note that the diffusion coefficient predicted by the analytical formula given in [2] is consistent with $D_0 \approx 13$ for a $\kappa \approx 13.7$, which is in reasonable agreement with our $\kappa \approx 12.5$.

From Eq. (3) we see that the quantum diffusion not only depends on the rate of decoherence as discussed in the previous paragraph, but also on D_0 and N^*. The main dependence is governed by the numerator, whereas the denominator is responsible for a saturation (*nonperturbative* regime [4]). The former critically depends on the degree of (classical) chaos as $N^* D_0 \sim \kappa^4$. We have measured this dependence by varying the pulse widths from 330 to 580 ns while leaving the detuning unchanged ($\delta_L/2\pi = 3.4$ GHz). This implies that the spontaneous scattering rate η changes as well, which, however, results in a weaker dependence according to $\eta \sim \kappa$. The experimental results are depicted in Fig. 3 together with the analytically calculated dependence. Again, we find good agreement, which confirms—albeit indirectly—the κ^4 dependence of the quantum diffusion rate. The physical significance of this dependence becomes obvious when writing Eq. (3) as $(\mathcal{D}_\infty/D_0)/\eta = N^* \sim (\kappa/\bar{k})^2$ (neglecting saturation effects). This quantity can be considered, loosely speaking, a measure for "vulnerability of quantum coherences" (the noise induced decoherence divided by the noise strength). Interestingly enough, the vulnerability is larger for a more classical system and also increases with an increasing degree of chaoticity in the corresponding C-DKR.

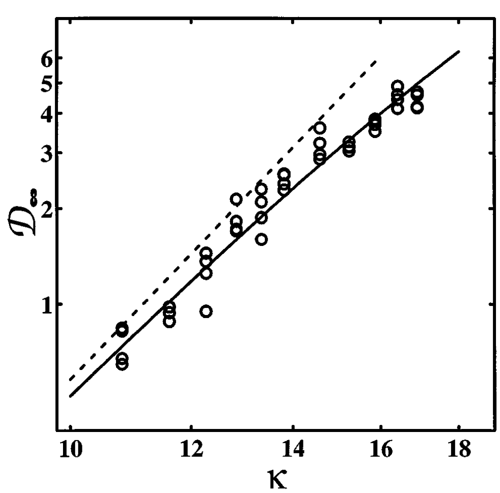

FIG. 3. The measured (circles) quantum diffusion coefficients $\mathcal{D}_\infty$ as a function of classical chaos parameter κ for $\bar{k} = 2.1$ and $\delta_{45}/2\pi = 3.4$ GHz. The lines show the analytical results with (solid line) and without (dashed line) saturation.

At this point, it is natural to ask to what extent the atoms behave as classical particles. Let us introduce a measure for classicality as $C \equiv t_E/t_c$, where t_E is the Ehrenfest time and $t_c \equiv 1/\eta$ is the lifetime for the quantum coherences. $C \gtrsim 1$ means that coherences are destroyed within the Ehrenfest time implying a classical-like behavior. Using the expressions in [4], we can write the classicality parameter as $C = \eta \ln(2\pi/\bar{k})/\ln(\kappa/2)$. As expected, C increases with a stronger coupling to the environment and with a decreasing Planck's constant. Somewhat surprising is the fact that a higher degree of chaos results in a lower classicality. Although this is not in contradiction with the arguments presented in the introduction (the more chaos, the stronger the quantum corrections make themselves felt), it seems strange that—by increasing κ—the quantum diffusion approaches the classical behavior and yet the degree of classicality *decreases.* This result reveals that there is *no* one-to-one correspondence between the ratio quantum/classical diffusion and classicality [10]. In conclusion, we would like to emphasize that by no means do we claim any degree of chaos in the presented quantum system. A coupling to an environment alone, i.e., without performing the semiclassical limit $\hbar \to 0$, does not render quantum dynamics *equal* to classical dynamics. Even if the noise level was so high that the energy diffusion looked essentially classical, the Wigner function representing the atomic dynamics would still *not* display sensitive dependence *nor* would it look anything like a point in a classical phase space. The often too lavishly used notion of "driving a quantum system back to classical behavior" only makes sense in the semiclassical limit.

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[10] We do not claim that this is generically true, e.g., for the DKR-p and the DKP models in Ref. [4] our conclusions do not hold.