A1. Well,

\[ 1 = \int_{1}^{\infty} cx^{-2} \, dx = c \lim_{b \to \infty} \left[ -x^{-1} \right]_1^b = c \lim_{b \to \infty} \left( -\frac{1}{b} + 1 \right) = c. \]

A2. As \( X \) varies from 1 to \( \infty \), \( Y \) varies from 0 to \( -\infty \). So the domain of \( f_Y \) is \( (-\infty, 0] \).

\[ F_Y(y) = P(Y \leq y) = P(-\log X \leq y) = P(X \geq e^{-y}) = 1 - F_X(e^{-y}) \]

\[ \Rightarrow f_Y(y) = \frac{d}{dy} (1 - F_X(e^{-y})) = -f_X(e^{-y}) \cdot (-e^{-y}) = e^{2y} e^{-y} = e^y. \]

[Notice that \( F_Y = f_Y \). It is then easy to check that \( \int_{-\infty}^{\infty} f_Y(y) \, dy = 1 \).]

B1. [This problem is a generalization of Exercise 4.38.] Using the definition of covariance and linearity of expectation, we compute

\[
\]

So the covariance is 0 exactly when \( X \) and \( Y \) have equal variance.

B2. We must check that \( P(X + Y = a, X - Y = b) = P(X + Y = a)P(X - Y = b) \) for all valid \( a, b \). [We cannot carry out this check, because we know so little about \( X \) and \( Y \).]

C. We model the time (in minutes) between arrivals using an exponentially distributed random variable \( X \) with parameter \( \lambda = 20 \). Then \( E[X] = 1/\lambda = 1/20 \) and \( V[X] = 1/\lambda^2 = 1/400 \).

The standard deviation is \( SD[X] = \sqrt{V[X]} = 1/20 \). As a rule of thumb, \( X \) will usually be in the interval \( E[X] \pm 2SD[X] \). In this case, the interval extends into the negative numbers,
indicating that it is not a great approximation. Keeping that caveat in mind, the time between arrivals will be around 3 seconds and usually between 0 seconds and 9 seconds.

**D1.** [Warning: Functions like this \( cx(x + y) \) appear in predator-prey studies, but not as probability density (mass) functions, to my knowledge. That is, the problem set-up is not scientifically authentic.] Well,

\[
P(X = x) = \sum_{y=1}^{n} P(X = x, Y = y) = \sum_{y=1}^{n} cx(x + y)
\]

\[
= cx^2 \sum_{y=1}^{n} 1 + cx \sum_{y=1}^{n} y
\]

\[
= cx^2 n + cxn(n + 1)/2
\]

and

\[
P(Y = y) = \sum_{x=1}^{m} P(X = x, Y = y) = \sum_{x=1}^{m} cx(x + y)
\]

\[
= cx \sum_{x=1}^{m} x^2 + cy \sum_{x=1}^{m} x
\]

\[
= cm(m + 1)(2m + 1)/6 + cym(m + 1)/2.
\]

**D2.** By the definition of conditional probability,

\[
P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} = \frac{x(x + y)}{m(m + 1)(2m + 1)/6 + ym(m + 1)/2}.
\]