Recall that

\[ \text{CLIQUE} = \{ \langle G, k \rangle : G \text{ is an undirected graph, } k \geq 1, \text{ and } G \text{ contains a } k\text{-clique} \}. \]

Also, for any \( k \geq 1 \), let

\[ \text{CLIQUE}_k = \{ \langle G \rangle : G \text{ is an undirected graph that contains a } k\text{-clique} \}. \]

In class, we will soon learn that \( \text{CLIQUE} \) is \( NP \)-complete. Without going into details, this means that if \( \text{CLIQUE} \in P \), then \( P = NP \). The common belief is that \( P \neq NP \), and hence \( \text{CLIQUE} \not\in P \).

A. Show that \( \text{CLIQUE}_k \in P \) for all \( k \). (For the sake of Problem B, it might help if you try to pin down your running time fairly precisely. By the way, the \( k = 3 \) case is Problem 7.9 in our textbook.)

B. Explain how it’s possible that \( \text{CLIQUE}_k \in P \) for all \( k \), but \( \text{CLIQUE} \not\in P \). In other words, explain why someone might think that \( (\forall k \text{ CLIQUE}_k \in P) \implies \text{CLIQUE} \in P \), and why that argument can’t be completed.