A. We’ve proven in class that $K(x)$ is not computable. So something must be wrong with the following argument. What’s wrong?

We will build a Turing machine $N$ that computes $K(x)$. Given an input $x$, $N$ tests all strings $y$, in lexicographic order, to see whether they are descriptions of $x$. For each $y$, $N$ first checks that $y$ is of the form $\langle M, w \rangle$. If it is, then $N$ runs $M$ on $w$. $N$ tests these strings $y$ in parallel, in the usual way: one step on the first string, then two steps on the first string and one on the second, etc. As soon as $N$ finds a string that describes $x$, $N$ halts with the length of that string on its tape.

This works because there is a bound on the length of string that $N$ must try. Let $M$ be the Turing machine that immediately halts, and let $c = |\langle M, \rangle|$. Then, for any string $x$, $\langle M, x \rangle$ is a description of $x$ of length $c + |x|$, and so $K(x) \leq c + |x|$. Thus $N$ will find a description of $x$ among the strings of length less than or equal to $c + |x|$.

B. Problem 6.23. (Hint: Mimic our proof that $K(x)$ is not computable.)

C. Prove that for any strings $x$ and $y$, $K(xy) \leq c + 2 \log_2 K(x) + K(x) + K(y)$. What exactly is $c$? (This problem is partially done in your book.)