Carleton College Math 111, Winter 2008, Exam 2

You have 60 minutes.

You may not use any notes or calculator.

Show your work and explain your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

If you have no idea how to solve a problem, or if you have forgotten a key formula that you think you need to know, you may ask me for a hint. The hint will cost you some points (to be decided unilaterally by me as I grade your paper), but it may help you earn more points overall.

Good luck.
1. Compute.
   A. $\int t^{17} \, dt$
   
   B. $\int x \sin(3x^2) \, dx$
   
   C. $\int \frac{e^{\sqrt{u}}}{\sqrt{u}} \, du$
2. Differentiate these functions.
   A. \( s(t) = \ln(\tan(t^2)) \)

B. \( f(x) = x^{(e^{2x}-1)} \)

3. After a lot of practice, I’ve become good at estimating \( e^a \) for any number \( a \). But still I have trouble computing \( \ln a \). Describe in detail a procedure, based on Newton’s method, that I can use to estimate \( \ln a \) for any positive number \( a \).
4. You are trying to persuade the government of British Columbia to purchase a 10,000 km$^2$ patch of land from the various people who currently own parts of it, to set it aside as a wildlife habitat. The more of it the government purchases, the better, because wild animals prefer large tracts of land far away from human influence; your research suggests that a preserve of $x$ km$^2$ could sustain $m(x) = \frac{1}{100}x^2$ large mammals. Unfortunately, the more land the government purchases, the more expensive each additional km$^2$ becomes; it seems that the cost of purchasing $x$ km$^2$ will be about $c(x) = \frac{25}{10,000 - x}$ (in thousands of Canadian dollars). How much land should the government buy, to maximize the number of large mammals protected per dollar?
5. Let \( B = \int_{-1}^{7} \sin(t^2) \, dt \).

A. What kind of object is \( B \) (e.g. a number, a function, a region in the plane, a point)? What is its geometric meaning?

B. Express \( B \) as a limit of Riemann sums. Be as detailed as possible.

C. Explain how you would find an approximate value for \( B \), if you were given a calculator.
6. In as much detail as possible, graph $y = xe^x$. 