Carleton College Math 111, Winter 2008, Exam 1

You have 60 minutes.

You may not use any notes or calculator.

Always show your work and explain all of your answers. Good work often earns partial credit. A correct answer with no explanation often earns little or no credit.

If you have no idea how to solve a problem, or if you have forgotten a key formula that you think you need to know, you may ask me for a hint. The hint will cost you some points (to be decided unilaterally by me as I grade your paper), but it may help you earn more points overall.

Good luck.
1. Differentiate the following functions. Do NOT simplify.
   A. \( y = 5x^6 - 2x^3 + 3x + 1 \).

   B. \( s(t) = \frac{t}{\cos t} \).

   C. \( p = 4^r \cdot (3r - 1) \).

   D. \( y = \left( 2e^{5t} + \sin \left( \sqrt{t} \right) \right)^7 \).
2. The Furtwängler Glacier on Mount Kilimanjaro is a giant sheet of ice. Suppose, for simplicity, that it is rectangular of length 600 m and width 100 m. Due to warming climate, its length and width are both shrinking by 5 m per year. Its thickness is a constant 6 m. How fast is the volume of the glacier changing? Simplify your answer, and include units.

3. Suppose that $x = f(t) = \cos(2t) + \sin(3t)$ is the position of a particle at time $t$. Compute the particle’s acceleration. Simplify and clearly mark your answer.
4. Recall that Newton’s law of cooling is expressed by the differential equation \( \frac{dy}{dt} = k(A - y) \).

A. Explain in words the physical meaning of these five quantities:

- \( t \)
- \( y \)
- \( A \)
- \( k \)
- \( \frac{dy}{dt} \)

B. We used \( k = 0.9 \) in the Santa-cooling problem in class. If we had used a lesser (but still positive) value of \( k \), such as 0.5, would Santa cool more slowly or more rapidly? Explain.

C. The value \( k = 0.9 \) was given to us by the police but not explained further. In words, describe some circumstances of Santa’s death that might influence the police estimate for \( k \).
5. Show that \( \lim_{h \to 0} h \sin \left( \frac{1}{h} \right) = 0. \)

6. Compute the derivative, at \( x = 0 \), of \( y = f(x) = \begin{cases} 
  x^2 \sin \left( \frac{1}{x} \right) & \text{if } x \neq 0, \\
  0 & \text{if } x = 0.
\end{cases} \)
7. The table below shows the number $s$ of web sites in existence, for each year $t$ in a 10-year period. I wish to model $s$ as a function of $t$. When I plot the data on a semilog plot, I get a line of slope 1.43 and intercept 2.31. What then is the function $s(t)$? Simplify your answer.

<table>
<thead>
<tr>
<th>year $t$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td># sites $s$</td>
<td>42</td>
<td>176</td>
<td>735</td>
<td>3070</td>
<td>12800</td>
<td>53600</td>
<td>224000</td>
<td>936000</td>
<td>3910000</td>
<td>16400000</td>
</tr>
</tbody>
</table>

8. Graph $y = e^{2x} - 4$ as precisely as you can (including the correct scale, intercepts, etc.).